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Review

Nonparametric estimation of the claim amount in the strong stability analysis of the classical risk model

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ABSTRACT

This paper presents an extension of the strong stability analysis in risk models using nonparametric kernel density estimation for the claim amounts. First, we detail the application of the strong stability method in risk models realized by V. Kalashnikov in 2000. In particular, we investigate the conditions and the approximation error of the real model, in which the probability distribution of the claim amounts is not known, by the classical risk model with exponentially distributed claim sizes. Using the nonparametric approach, we propose different kernel estimators for the density of claim amounts in the real model. A simulation study is performed to numerically compare between the approximation errors (stability bounds) obtained using the different proposed kernel densities.

Contents

1. Introduction

In ruin theory, stochastic processes are used to model the surplus of an insurance company and to evaluate its ruin probability, i.e., the probability that the total amount of claims exceeds its reserve. This characteristic is a much studied risk measure in the literature. In general, this measure in finite and in infinite time is very difficult or even impossible to evaluate explicitly. Thus, different approximation methods have been proposed to estimate this characteristic (see Asmussen and Albrecher, 2010; Grandell, 1990).

We consider throughout this paper the two risk reserve processes $\{S(t), t \geq 0\}$ and $\{S'(t), t \geq 0\}$ which are given by:

$$S(t) = u + ct - \sum_{i=1}^{N(t)} Z_i, \quad t \geq 0, \tag{1}$$

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$$S'(t) = u + ct - \sum_{i=1}^{N(t)} Z'_i, \quad t \geq 0, \tag{2}$$

where $u \geq 0$ is the initial reserve, $c > 0$ represents the premium rate and $\{N(t), t \geq 0\}$ is a Poisson process with parameter λ. The independent and identically distributed random variables of claim amount $\{Z_i\}_{i\in\mathbb{N}^*}$ and $\{Z'_i\}_{i\in\mathbb{N}^*}$ have distinct distribution functions F and F'.

The question of stability in actuarial risk theory naturally arises for two principal reasons. First, the parameters that govern the risk model are obtained using statistical methods. Second, the ruin probability:

$$\Psi(u) = \mathbb{P}(\exists t \geq 0 | S(t) < 0), \quad \forall\, u \geq 0, \tag{3}$$

cannot be found explicitly. Hence, it is necessary to obtain explicit stability bounds. The strong stability method, which was developed by Aissani and Kartashov (1983), makes it possible to clarify the conditions for which the ruin probability of the complex risk model (real model) defined by the process (1) can be approximated by the corresponding ruin probability in the simple risk model (ideal model) defined by the process (2). In other words, the model defined by the process (2) may be used as a good approximation or idealization of the real model defined by the process (1).

With a certain norm $\|\cdot\|_v$, Kalashnikov (2000) presented a new stability bound for the ruin probability which has the following form:

$$\|\Psi - \Psi'\|_v \leq \Pi(\|F - F'\|_v, c, \lambda), \tag{4}$$

where Π is a function continuous at 0 with $\Pi(0) = 0$.

In this sense, further work was done for other models: the risk model with investment (Rusaityte, 2001), semi-Markov risk models (Enikeeva et al., 2001) and the two-dimensional classical risk model (Benouaret and Aissani, 2010).

The strong stability analysis is part of the robustness theory, i.e., when we do not know exact values of the model parameters (inputs), it is natural to measure the impact of a small perturbation of the model on the outputs. The influence function, which was used by Marceau and Rioux (2001), Loisel et al. (2008) in the sensitivity and robustness analysis of ruin probabilities, is one of the tools to measure this impact. However, strong stability is another tool to measure the deviation between the ruin probabilities. In contrast to the influence function, this technique based on the disturbance of a linear operator permits us to investigate the ergodicity and the stability of the stationary and non-stationary characteristics of Markov chains (see Aissani and Kartashov, 1984).

There is an alternative method for computing the bounds on the perturbations of Markov chains closely related to the strong stability approach which is the series expansion approach for Markov chains (see Hamoudi et al., 2014). In contrast to the strong stability method, the series expansion approach requires numerical computation of the deviation matrix, which limits the approach to Markov chains with a finite state space (see Heidergott et al., 2010,b).

For a theoretical study, different probability laws can be used to model the amount of claims. In practice, the determination of these probability distributions requires the use of functional estimation techniques (see Bareche and Aissani, 2008, 2010; Zhang et al., 2014). Our contribution in this work is to use the nonparametric estimation of the claim amounts in the strong stability analysis of the ruin probabilities. Assume that the law of the claim amounts is exponential in the ideal risk model described by the process $S'(t)$ and the law of the claim amounts is general in the real risk model described by the process $S(t)$. We clarify, using the strong stability method, the conditions for approximating ruin probabilities Ψ by Ψ' and we estimate the error of this approximation given in bound (4).

This paper is organized as follows: in Section 2, we give the basics of strong stability method applied to the classical risk models. In Section 3, we present some kernels proposed in nonparametric estimation of the density of claim amounts. The main results of this paper are presented and numerically illustrated in Section 4.

2. Preliminaries and position of the problem

In this section, we present some necessary notations, the basic theorems of the strong stability method and the theoretical results obtained by applying this method in the risk models (see Kartashov, 1996; Kalashnikov, 2000; Benouaret and Aissani, 2010).

2.1. The strong stability criteria

We denote by $m\varepsilon$ the space of finite measures on the probabilizable space (E, ε), and we introduce the special family of norms defined by:

$$\|m\|_v = \int_E v(x)|m|(dx), \quad \forall\, m \in m\varepsilon, \tag{5}$$

where v is a measurable function that is bounded below away from zero (not necessarily finite).

This norm induces, in the space $f\varepsilon$ of bounded measurable functions on E, the norm:

$$\|f\|_v = \sup\{|mf|, \|mf\|_v \leq 1\} = \sup_{x\in E}[v(x)]^{-1}|f(x)|, \quad \forall f \in f\varepsilon \tag{6}$$

The norm of the transition kernel P in the space β is given as follows:

$$\|P\|_v = \sup_{x\in E}\left([v(x)]^{-1}\int_E v(y)|P(x, dy)|\right), \tag{7}$$

where β is the space of linear operators.

Definition 2.1 (*see Aissani and Kartashov, 1983*)**.** The Markov chain X with transition kernel P and stationary distribution π is said to be v-strongly stable with respect to the norm $\|.\|_v$ if $\|P\|_v < \infty$ and each stochastic kernel Q in the neighborhood $\{Q : \|Q - P\|_v < \epsilon\}$ has a unique invariant measure $\pi' = \pi'(Q)$ and $\|\pi - \pi'\|_v \to 0$ as $\|Q - P\|_v \to 0$.

The following theorem was proved by Kartashov (1996) and was applied in a risk model with one line of business by Kalashnikov (2000).

Theorem 2.1. *Let v be a fixed weight function. Consider a Markov chain with transition kernel P, such as $\|P\|_v < \infty$, and that has a unique stationary distribution π. Additionally, suppose that there is a non-negative function h and a probability measure α such that P can be decomposed as follows:*

$$P(u, \cdot) = T(u, \cdot) + h(u)\,\alpha(\cdot), \tag{8}$$

where

$$\|\pi\|_h > 0, \qquad \|\alpha\|_h > 0, \tag{9}$$

and

$$\|T\|_v \leq \rho < 1. \tag{10}$$

Then, all Markov chains with transition kernel P' that satisfies:

$$\Delta = \|P - P'\|_v < \Delta_0 \equiv \frac{(1-\rho)^2}{1 - \rho + \rho\,\|\alpha\|_v}, \tag{11}$$

have a unique stationary distribution π', and furthermore,

$$\|\pi - \pi'\|_v \leq \frac{\Delta \|\alpha\|_v}{(1-\rho)(\Delta_0 - \Delta)}. \tag{12}$$

2.2. The strong stability of the classical risk model

From applying the qualitative aspect of the strong stability method, we have the strong stability of the classical risk model, which means that a small deviation of its parameters leads to a small deviation of its characteristics (see Benouaret and Aissani, 2010).

In this part, we use the quantitative aspect of the strong stability method to estimate the approximation error of the real risk model by the classical risk model.

Denote by $a = (\lambda, c, F)$ (respectively $a' = (\lambda', c', F')$) the vector parameter of the real risk model (respectively of the ideal risk model).

The estimation of the transition kernel deviation, obtained by Kalashnikov, is given by the following formula:

$$\|P - P'\|_v \leq 2\,\mathbb{E}\, e^{\epsilon Z} \left| \ln \frac{\lambda c'}{\lambda' c} \right| + \|F - F'\|_v. \tag{13}$$

Under the following condition, which represents the perturbation domain of the parameters,

$$u(a, a') \leq (1-\rho)^2, \tag{14}$$

where

$$u(a, a') = 2\,\mathbb{E}\, e^{\epsilon Z} \left| \ln \frac{\lambda c'}{\lambda' c} \right| + \|F - F'\|_v, \tag{15}$$

we have the following strong stability inequality:

$$\|\Psi - \Psi'\|_v \leq \frac{\mu(a, a')}{(1-\rho)\ \left((1-\rho)^2 - \mu(a, a')\right)}, \tag{16}$$

where $\rho(\epsilon) = \mathbb{E} \exp(\epsilon(Z - c\,\theta))$ and θ is a random variable that represents the inter-arrival of claims.

Denote by Γ the upper bound given by the inequality (16),

$$\Gamma = \frac{\mu(a, a')}{(1-\rho)\ \left((1-\rho)^2 - \mu(a, a')\right)}. \tag{17}$$

Specific case: Perturbation of the claim sizes

In our study, we take into consideration only the perturbation of claim amounts. The other parameters, λ and c, are the same for both models (ideal and real).

In this case, the transition kernel deviation given by the inequality (13) becomes:

$$\|P - P'\|_v \leq \|F - E_\mu\|_v, \tag{18}$$

where F is the unknown distribution of the claim amounts in the real model and E_μ is the exponential distribution of the claim amounts in the ideal model.

Therefore, we obtain the following bound of stability:

$$\|\Psi - \Psi'\|_v \leq \frac{\|F - E_\mu\|_v}{(1-\rho)\ \left((1-\rho)^2 - \|F - E_\mu\|_v\right)}. \tag{19}$$

Since the distribution of the claim sizes F is unknown, its density function must be approximated using the nonparametric density estimation.

3. Kernel density estimation of the claim amount

The kernel density method is much commonly used in the nonparametric estimation methods. Thus, when we have an independent and identically distributed sample $X_1, \ldots, X_n$ coming from a random variable X with an unknown probability density function f on $\Re \subseteq \mathbb{R}$, the associated kernel estimators, asymmetric and continuous, have the following form:

$$f_h(x) = \frac{1}{n} \sum_{i=1}^{n} K_{x,h}(X_i), \tag{20}$$

where h is the smoothing parameter and $K_{x,h}$ is the asymmetric kernel.

3.1. The choice of kernel

The density function of the claim amounts is defined on a bounded support. In order to avoid the problem of boundary effects, due to the use of symmetric kernel (see Bareche and Aissani, 2010), we use the following asymmetric kernel, which never assigns a weight out of the support.

3.1.1. Modified gamma kernel

The Gamma kernel estimator was introduced by Chen (2000) for a probability density function that has a bounded support on $\Re = [0, \infty[$.

Two classes of kernels have been proposed:

$$K_{GAM\left(\frac{x}{h}+1,h\right)}(u) = \frac{u^{\frac{x}{h}} \exp\left(\frac{-u}{h}\right)}{h^{\frac{x}{h}+1}\Gamma\left(\frac{x}{h}+1\right)}, \tag{21}$$

where $\Gamma(\alpha) = \int_0^\infty \exp(-t) t^{\alpha-1} dt$, and the associated estimator is given as follows:

$$f_h^{GAM}(x) = \frac{1}{n} \sum_{i=1}^{n} K_{\frac{x}{h}+1,h}(X_i). \tag{22}$$

The second class is the modified Gamma kernel, which is given as follows:

$$K_{GAM1(\rho_h(x),h)}(u) = \frac{u^{\rho_h(x)-1} \exp\left(\frac{-u}{h}\right)}{h^{\rho_h(x)}\Gamma(\rho_h(x))}, \tag{23}$$

where

$$\rho_h(x) = \begin{cases} \dfrac{x}{h} & \text{if } x \geq 2h, \\ \dfrac{1}{4}\left(\dfrac{x}{h}\right)^2 + 1 & \text{if } 0 \leq x < 2h. \end{cases} \tag{24}$$

The estimator with a Gamma-modified kernel is given by:

$$f_h^{GAM1}(x) = \frac{1}{n} \sum_{i=1}^{n} K_{\rho_h(x),h}(X_i). \tag{25}$$

Malec and Schienle (2012) proposed an improvement for the function ρ_h.

$$\rho_h^1(x) = \begin{cases} \left[\dfrac{1}{4}\left(\dfrac{x}{hr}\right)^2 + 1\right][r + 2h(1-r)] & \text{si } 0 \leq x < 2hr, \\ \dfrac{x}{hr}(r + 2h - x) & \text{si } 2hr \leq x < 2h, \\ \dfrac{x}{h} & \text{si } x \geq 2h, \end{cases} \tag{26}$$

where $r \in]0, 1]$ and for $r = 1$, we return to the standard Gamma kernel.

The new estimator with a Gamma kernel is given as follows:

$$f_h^{GAM2}(x) = \frac{1}{n} \sum_{i=1}^{n} K_{(\rho_h^1(x),h)}(X_i). \tag{27}$$

3.1.2. Reciprocal inverse Gaussian kernel

To realize a comparison with the Gamma kernel, we use the reciprocal inverse Gaussian kernel (RIG) (see Scaillet, 2004), which has the following form:

$$K_{RIG\left(\frac{1}{x-h},\frac{1}{h}\right)}(u) = \frac{1}{\sqrt{2\pi hu}} \exp\left(\frac{-(x-h)}{2h}\left(\frac{u}{x-h} - 2 + \frac{x-h}{u}\right)\right) \tag{28}$$

and its associated estimator is given by the following equation:

$$f_h^{RIG}(x) = \frac{1}{n}\sum_{i=1}^{n} K_{RIG\left(\frac{1}{x-h},\frac{1}{h}\right)}(X_i). \tag{29}$$

3.2. The choice of the smoothing parameter

In the literature, several methods to select the parameter have been proposed. The first class of methods, called plug-in, was proposed by Woodroof (1970). The second class, based on cross-validation, was proposed by Hermans et al. (1974). Another method using the Bayesian approach (see Zougab et al., 2013), is applied when the sample size is moderate.

In this work, the smoothing parameter h is chosen to minimize the criterion of the least squares cross-validation (see Rudemo, 1982; Bowman, 1984). The optimal smoothing parameter is obtained as follows:

$$h_{ucv} = arg\min_h UCV(h), \tag{30}$$

where

$$UCV(h) = \int_{\mathbb{R}} f_h^2(x)dx - \frac{2}{n}\sum_{i=1}^{n} f_{h,i}(X_i),$$

with

$$f_{h,i}(X_i) = \frac{1}{(n-1)h}\sum_{j=1,j\neq i}^{n} K_{(X_i,h)}(X_j).$$

4. Numerical evaluation for the strong stability bound

In this section, we want to apply the kernel density method to numerically estimate the approximation error between the proposed models by evaluating the bound of the transition kernel deviation given in (18) and the bound of the ruin probability deviation given in (19).

For this purpose, we develop an algorithm that contains the following steps:

4.1. Algorithm

1. Generate a sample of size n from a general distribution of the claim amounts;
2. Use the different kernels given in Section 3 to estimate the density function f by f_h;
3. Introduce the average arrival rate of the claims λ and the premium rate c;
4. Evaluate the average claim amount $\mu \leftarrow \int_0^\infty x f_h(x)$;
5. Verify if $c > \lambda\mu$, otherwise, the ruin is certain;
6. Determine the field of ϵ, such as $\epsilon_{min} < \epsilon < \epsilon_{max}$, where ϵ_{min} (respectively ϵ_{max}) is the smallest value (respectively the largest value), which verifies the two following conditions:

$$0 < \epsilon < min\left\{\frac{1}{\mu}, \frac{c-\lambda\mu}{cu}\right\} \quad \text{and} \quad u(a,a') < (1-\rho(\epsilon))^2;$$

7. Determine the approximation error $\Gamma = \frac{u(a,a')}{(1-\rho)((1-\rho)^2 - u(a,a'))}$.

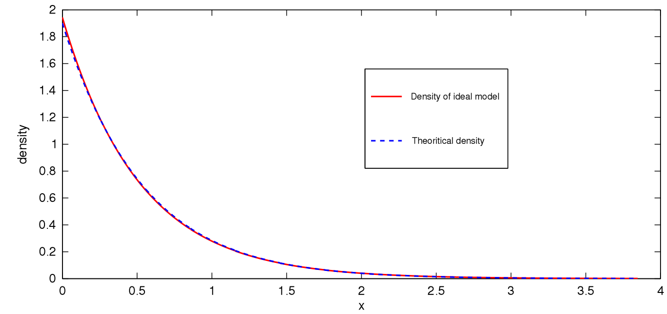

Fig. 1. Theoretical density $f_1(Cox(2,3,0.05))$ and exponential density.

4.2. Numerical examples

For this numerical study, we generate a sample of size n from a general distribution of the claim amounts to be able to estimate the probability density f_h using the kernel density method. In this numerical example, the general distributions considered are the Cox law ($Cox2(\mu_1 = 2, \mu_2 = 3, \alpha = 0.05)$) and the Weibull law ($Weibull(\delta = 1.2, \beta = 2)$), whose densities are, respectively, given as follows:

$$f_1(x) = \begin{cases} (1-\alpha)\mu_1 e^{-\mu_1 x} + \dfrac{\alpha\mu_2}{\mu_2-\mu_1}\mu_1 e^{-\mu_1 x} \\ \quad + \dfrac{\alpha\mu_1}{\mu_1-\mu_2}\mu_2 e^{-\mu_2 x} \quad \text{if } x \geq 0 \text{ and } \mu_1 \neq \mu_2; \\ (1-\alpha)\mu_1 e^{-\mu_1 x} + \alpha\mu_1 e^{-\mu_1 x} \quad \text{if } x \geq 0 \text{ and } \mu_1 = \mu_2; \\ 0 \quad \text{otherwise} \end{cases}$$

and

$$f_2(x) = \begin{cases} \delta\beta x^{\delta-1} e^{-\beta x^\delta} & \text{if } x > 0; \\ 0 & \text{otherwise.} \end{cases}$$

For the two models (ideal and real), we fix the average arrival rate of claims $\lambda = 0.1$, the premium rate $c = 5$, the sample size $n = 200$ and the number of simulations $R = 100$.

Using the environment MATLAB 7.4 (R2007a), the results of this simulation are presented in Tables 1 and 2. The curve of the theoretical density f_1 (respectively f_2) and the curve of the exponential density are presented in Fig. 1 (respectively in Fig. 3).

The curve of the theoretical density f_1 (respectively f_2) and the estimated densities are given in Fig. 2(respectively in Fig. 4).

Discussion. From Fig. 1, we observe that the density curve of the ideal model is closer to the curve of the theoretical density compared to those in Fig. 3. In other words, the perturbation of the ideal model (perturbation of claim amounts) in the first case (with density $f1$ $Cox(2,3,0.05)$) is smaller than the perturbation of the ideal model in the second case (with density $f2$ $Weibull(1.2,2)$).

In Table 1, we observe that the approximation error for the ruin probabilities of the real and ideal models using the estimator with the GAM2 kernel ($\Gamma = 0.0404$) is the closest to the one obtained using the theoretical density f_1 ($\Gamma = 0.0171$).

However, in Table 2, the approximation error using the theoretical density f_2 ($\Gamma = 0.3337$) is higher than those obtained using the GAM kernel estimator ($\Gamma = 0.2816$).

4.3. Variation of error Γ in function of ϵ

The evolution of the error Γ in function of the norm parameter ϵ with theoretical densities $f_1(Cox(2,3,0.05))$ and $f_2(Weibull(1.2,2))$ are presented in Fig. 5.

Table 1
Strong stability bound of ruin probability with different estimators and with theoretical density f_1.

	f_1	f_{1h}^{GAM}	f_{1h}^{GAM1}	f_{1h}^{GAM2}	f_{1h}^{RIG}
Mean claim amount μ	0.5145	0.4973	0.5193	0.5146	0.5079
$0 < \epsilon < min\{\frac{1}{\mu}, \frac{c-\lambda\mu}{c\mu}\}$	]0, 1.9237[	]0, 1.9907[	]0, 1.9056[	]0, 1.9232[	]0, 1.9489[
$\|F_1 - E_{\frac{1}{\mu}}\|_v$	0.0133	0.0376	0.0342	0.0309	0.0358
Γ	0.0171	0.0495	0.0424	0.0404	0.0470

Table 2
Strong stability bound of ruin probability with different estimators and with theoretical density f_2.

	f_2	f_{2h}^{GAM}	f_{2h}^{GAM1}	f_{2h}^{GAM2}	f_{2h}^{RIG}
Mean claim amount μ	0.5268	0.5280	0.5479	0.5479	0.5381
$0 < \epsilon < min\{\frac{1}{\mu}, \frac{c-\lambda\mu}{c\mu}\}$	]0, 1.8784[	]0, 1.8741[	]0, 1.8051[	]0, 1.8052[	]0, 1.8385[
$\|F_2 - E_{\frac{1}{\mu}}\|_v$	0.1986	0.1739	0.1978	0.2005	0.2038
Γ	0.3337	0.2816	0.3359	0.3418	0.3457

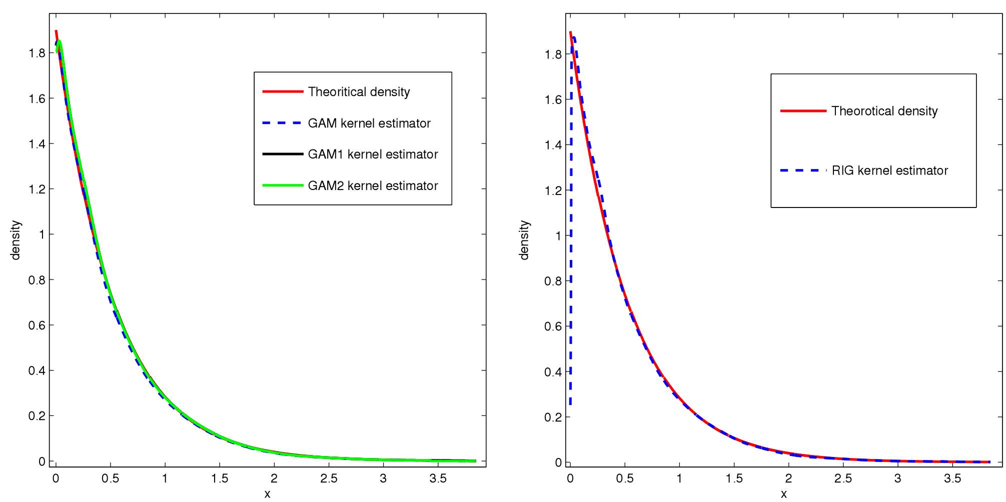

Fig. 2. Theoretical density $f_1(Cox(2, 3, 0.05))$ and its estimated densities.

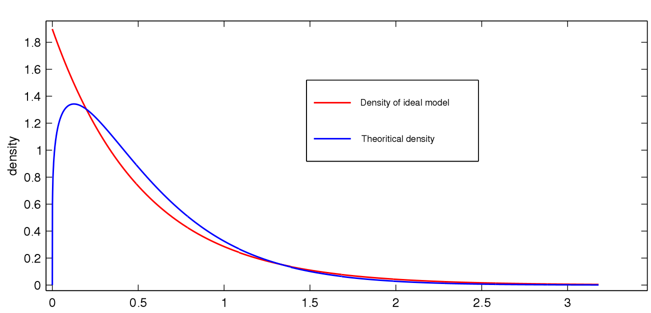

Fig. 3. Theoretical density $f_2(Weibull(1.2, 2))$ and exponential density.

Fig. 6 (respectively Fig. 7) describes the evolution of the error Γ in function of ϵ with theoretical density $f_1(Cox(2, 3, 0.05))$ (respectively $f_2(Weibull(1.2, 2))$) and with its estimated densities.

Discussion. Note, according to Fig. 5 with theoretical density $f_1(Cox(2, 3, 0.05))$ (resp. $f_2(Weibull(1.2, 2))$), that the error, being important at the start, decreases speedily for the values of ϵ in the neighborhood of the lower bound. This may be explained by the fact that they are at the boundary of the stability domain (critical region). We notice also that the error increases speedily in the neighborhood of the upper bound (critical region). In contrast, everywhere else, the error increases reasonably with the values of ϵ.

5. Conclusion

In this work, we have developed a nonparametric study in the strong stability analysis of the classical risk model. For this purpose, we have exploited four types of kernels to estimate the unknown probability distribution of the claim amounts, and we have determined the strong stability bound of the ruin probability with each kernel.

From the realized simulation approach, the obtained numerical results are significant to the concept of the strong stability method, where for a small deviation of the parameters, we have a small deviation of the characteristics (ruin probability).

Moreover, compared to the stability bounds obtained with the theoretical density, we conclude that if the perturbation is very small, we can decide about the numerical comparison between the kernels estimators proposed for the claim amounts. In other words, a good estimator of the claim amounts distribution is one that has the smallest strong stability bound. Conversely, if the perturbation is important, it is not sufficient to select a good estimator.

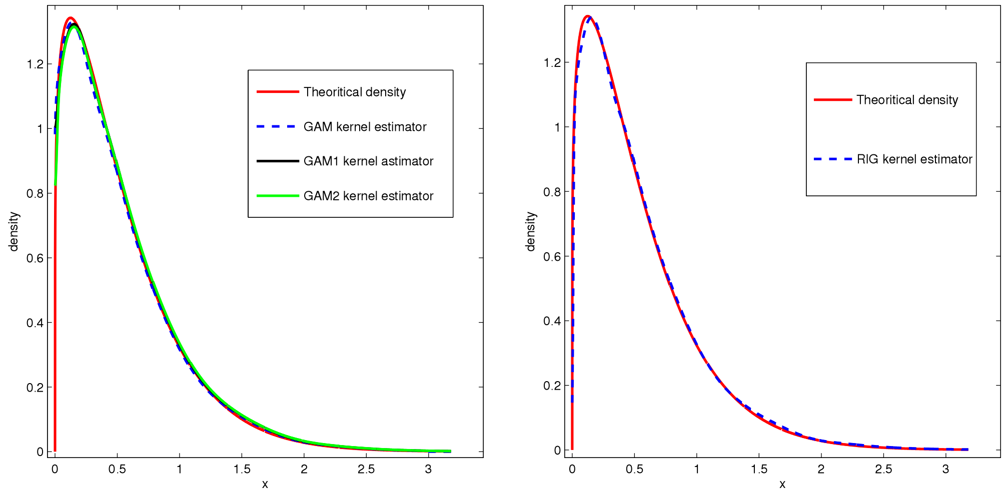

Fig. 4. Theoretical density $f_2(Weibull(1.2, 2))$ and its estimated densities.

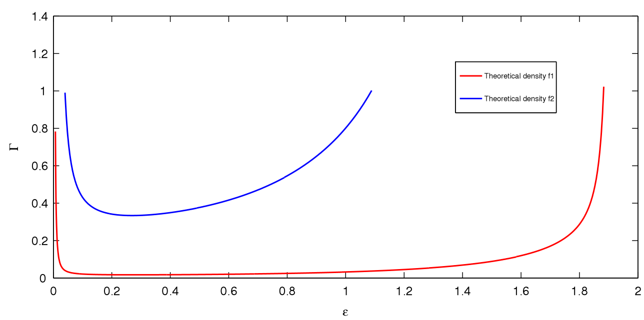

Fig. 5. Error Γ in function of ϵ with theoretical density f_1 and theoretical density f_2.

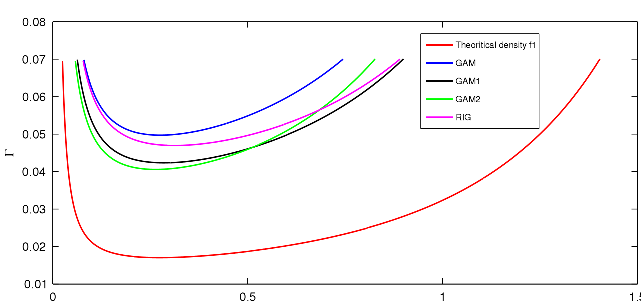

Fig. 6. Error Γ in function of ϵ with theoretical density f_1 and with its different estimators.

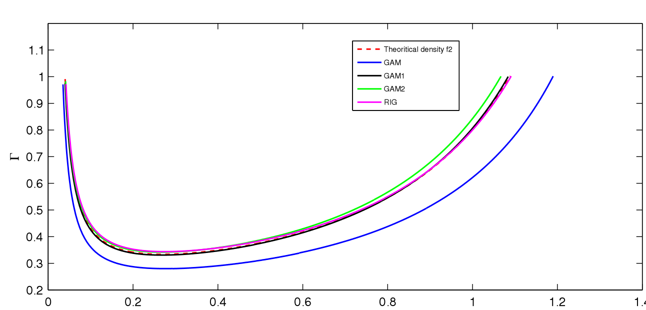

Fig. 7. Error Γ in function of ϵ with theoretical density f_2 and with its different estimators.

References

Aissani, D., Kartashov, N.V., 1983. Ergodicity and stability of Markov chains with respect to operator topology in the space of transition kernels. Dokl. Akad. Nauk Ukr. Ser. A 11, 3–5.

Aissani, D., Kartashov, N.V., 1984. Strong stability of an imbedded Markov chain in an M/G/1 system. Theor. Probab. Math. Stat. 29, 1–5.

Asmussen, S., Albrecher, H., 2010. Ruin Probabilities. Word Scientific.

Bareche, A., Aissani, D., 2008. Kernel density in the study of the strong stability of the M/M/1 queuing system. Oper. Res. Lett. 36 (5), 535–538.

Bareche, A., Aissani, D., 2010. Statistical techniques for a numerical evaluation of the proximity of G/G/1 and G/M/1 queuing systems. Comp. Math. Appl. 61, 1296–1304.

Benouaret, Z., Aissani, D., 2010. Strong stability in a two-dimensional classical risk model with independent claims. Scand. Actuar. J. 2, 83–93.

Bowman, A.W., 1984. An alternative method of cross-validation for the smoothing density estimates. Biometrika 71, 353–360.

Chen, SX., 2000. Gamma kernel estimators for density functions. Ann. Inst. Statist. Math. 52, 471–480.

Enikeeva, F., Kalashnikov, V., Rusaityte, D., 2001. Continuity estimates for ruin probabilities. Scand. Actuar. J. 1, 18–39.

Grandell, J., 1990. Aspect of Risk Theory. Springer-Verlage.

Hamoudi, Z., Aissani, D., Heidergott, B., 2014. Non-parametric sensitivity analysis of the finite M/M/1 queue. IFAC Proc. Vol. 47 (2), 498–503.

Heidergott, B., Hordijk, A., Leder, N., 2010a. Series expansions for continuous-time Markov processes. Oper. Res. 58 (3), 756–767.

Heidergott, B., Hordijk, A., Van Uitert, M., 2010b. Series expansions for finite-state Markov chains. Probab. Engrg. Inform. Sci. 21, 381–400.

Hermans, J.W., Habbema, J.D.F, Vandenbroek, K., 1974. A stepwise discriminant analysis program using density estimation. In: Compstat 1974. Bruckmann, pp. 101–110.

Kalashnikov, V., 2000. The stability concept for stochastic risk models, Working Paper Nr 166. Lab. of Actuarial Mathematics. University of Copenhagen.

Kartashov, N.V., 1996. Strong Stable Markov Chains. VSP, Utrecht.

Loisel, S., Mazza, C., Rullière, D., 2008. Robustness analysis and convergence of empirical finite-time ruin probabilities and estimation risk solvency margin. Insurance Math. Econom. 42 (2), 746–762.

Malec, P., Schienle, M., 2012. Nonparametric kernel density estimation near the boundary, SFB 649 'Economic Risk' Berlin.

Marceau, E., Rioux, J., 2001. On robustness in risk theory. Insurance Math. Econom. 29, 167–185.

Rudemo, M., 1982. Empirical choice of histograms and kernel density estimators. Scand. J. Stat. 9, 65–78.

Rusaityte, D., 2001. Stability bounds for ruin probabilities in a Markov modualited risk model with investments, Working Paper Nr 178. Lab. of Actuarial Mathematics, University of Copenhagen.

Scaillet, O., 2004. Density estimation using inverse and reciprocal inverse gaussian kernels. J. Nonparametr. Stat. 16, 217–226.

Woodroof, M., 1970. On choosing a delta sequence. Ann. Math. Stat. 41, 1665–1671.

Zhang, Z., Yang, H., Yang, H., 2014. On a nonparametric estimator for ruin probability in the classical risk model. Scand. Actuar. J. 4, 309–338.

Zougab, N., Adjabi, S., Kokonendji, C.C., 2013. A bayesian approch to bandwith selection in univariate associate kernel estimation. J. Stat. Theory Pract. 7 (1), 8–23.