ISSN 0321-3900

Theory of Stochastic Processes

10(26) • 2004 № 3-4

scientific
TBiMC
publishers

Theory of Stochastic Processes
Vol. 10 (26), no. 3–4, 2004, pp. 129–135

UDC 519.21

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STABILITY ANALYSIS IN AN INVENTORY MODEL

In this paper, we make a synthesis about the stability analysis in inventory models. In particular, we prove the applicability of the strong stability method to inventory models. We show the strong v−stability of the embedded Markov chain in an (R, s, S) periodic review inventory model with respect to the disturbance of the demand distribution. The case of the (R, S) model is also highlighted.

Introducton

F.W. Harris's paper [6] published in 1913 was the first contribution to inventory research. Harris has considered a deterministic case. Stochastic inventory models, in witch we consider demand as a stochastic process, were introduced in early 1950s (see [2]). Since, thousands of papers have treated this subject and this field still a wide area for research. For a recent reference we suggest [10].

A stochastic inventory model can include a large number of parameters, each can take many values and lot of them can be stochastic. Inventory problems are often very complicated and we need to use approximations to resolve them. Therefore, it is very important to justify these approximations and to estimate the resultant error. Furthermore, model parameters are imprecisely known because they are obtained from empirical data by statistical methods. This show the importance of the study of the stability because this can help us to test the sensitivity of the mathematical model to perturbations and to see whether or not it is a good representation of the real system.

Inventory models are the first stochastic models for witch monotonicity properties where proved (see [7, 11]). In 1969, Boylan have proved a robustness theorem for inventory problems by showing that the solution of the optimal inventory equation depends continually on its parameters including the demand distribution [4]. Recently, Chen and Zheng (1997) have showed that the inventory cost in an (R, s, S) model is relatively insensitive to the changes in $D = S - s$ [5].

In this paper, we apply the strong stability method developed in early 1980s [1]. In addition to the qualitative affirmation of the continuity, the strong stability method allows us to obtain quantitative estimates with an exact computation of constants. For basic definitions and results on this method, see for example [9].

In section (1), we describe our model. In section (2), we introduce notations, define the strong stability criterion and give a sufficiency theorem [1]. The main results of this paper are presented in section (3). Finally, we give in conclusion some research perspectives.

2000 *AMS Mathematics Subject Classification.* Primary 60J50.

Key words and phrases. Inventory control, (R, s, S) policy, (R, S) policy, Markov chain, Perturbation, Strong Stability.

1. Model description

Consider the following single-item, single-echelon, periodic review inventory model. Inventory level is inspected every R time units and if, at review moment, the inventory level is bellow or equal to s we order so much items to raise the inventory level to S. Suppose that orders arrives immediately. In period n, total demand is ξ_n. Assume that ξ_n, $n = 1, 2, ...$ are independent and identically distributed random variables with common probabilities,

$$a_k = P(\xi_1 = k), \quad k = 0, 1, ...$$

Also, denote by F_ξ their common distribution function. This model can be found in the earlier stochastic inventory literature (see for example [2]).

A special case of this model is the well known (R, S) model which correspond to the case $s = S - 1$. In the (R, S) system, the inventory level is inspected every R time units, and, if necessary, we order so much items to raise the inventory level to S.

Consider another (R, s, S) inventory model with the same structure, but with demands ξ'_n, $n = 1, 2, ...$ having common probabilities

$$a'_k = P(\xi'_1 = k), \quad k = 0, 1, ...$$

Let $(X_n)_{n\geq 1}$ and $(X'_n)_{n\geq 1}$ the Markov chains representing the on-hand inventory level at the end of periods in the two models and denote by P and Q their respective transition operators and by $E = \{0, 1, ..., S\}$ their common state space.

Also, denote by $(Y_n)_{n\geq 1}$ and $(Y'_n)_{n\geq 1}$ the Markov chains representing the on-hand inventory level at the end of periods in the two models when using an (R, S) ordering policy.

2. Strong stability criterion

Let $X = (X_t, t \geq 0)$, a homogeneous Markov chain with values in a measurable space $(E, \mathfrak{E})$, (where we assume that the σ-algebra $\mathfrak{E}$ is denumerably engendered), given by a regular transition kernel $P(x, A)$, $x \in E$, $A \in \mathfrak{E}$ and having a unique invariant measure π.

Denote by $m\mathfrak{E}$ $(m\mathfrak{E}^+)$ the space of finite (nonnegative) measures on $\mathfrak{E}$, $f\mathfrak{E}$ $(f\mathfrak{E}^+)$ the space of bounded measurable (nonnegative) functions on E.

Consider in the space $m\mathfrak{E}$, the Banach space $\mathfrak{M} = \{\mu \in (m\mathfrak{E}) : \|\mu\| < \infty\}$ with norm $\|.\|$ compatible with the structural order in $m\mathfrak{E}$, i.e. :

$$\|\mu_1\| \leq \|\mu_1 + \mu_2\| \text{ for } \mu_i \in \mathfrak{M}^+, i = 1, 2.$$
$$\|\mu_1\| \leq \|\mu_1 - \mu_2\| \text{ for } \mu_i \in \mathfrak{M}^+, i = 1, 2 \text{ and } \mu_1 \perp \mu_2.$$
$$|\mu|(E) \leq k\|\mu\| \text{ for } \mu \in \mathfrak{M}.$$

where $|\mu|$ is the variation of the measure μ, k is a finite positive constant and $\mathfrak{M}^+ = \mathfrak{M} \cap (m\mathfrak{E}^+)$.

We introduce in $m\mathfrak{E}$, the special family of norms

$$\|\mu\|_v = \int_E v(x)|\mu|(dx), \forall \mu \in m\mathfrak{E}$$

where v is a measurable function bounded from bellow by a positive constant, (not necessary finite) on E.

So, the induced norms on spaces $f\mathfrak{E}$ and $\mathfrak{M}$ will have the following forms :

$$\|P\|_v = \sup\{\|\mu P\|_v, \|\mu\|_v \leq 1\} = \sup_{x\in E} (v(x))^{-1} \int_E |P(x, dy)|v(y),$$
$$\|f\|_v = \sup\{|\mu f|, \|\mu\|_v \leq 1\} \quad = \sup_{x\in E} (v(x))^{-1} |f(x)|.$$

We associate to every transition kernel $P(x, A)$ in the space of bounded linear operators, the linear mappings $\mathfrak{L}_P : m\mathfrak{E} \to m\mathfrak{E}$ and $\mathfrak{L}_P^* : f\mathfrak{E} \to f\mathfrak{E}$, witch values for $\mu \in m\mathfrak{E}$ and $f \in f\mathfrak{E}$ are respectively :

$$\mu P(A) = \mathfrak{L}_P(\mu)(A) = \int_E \mu(dx)P(x, A), \quad \forall A \in \mathfrak{E},$$

$$Pf(x) = \mathfrak{L}_P^*(f)(x) = \int_E P(x, dy)f(y), \quad \forall x \in E.$$

and to every function $f \in f\mathfrak{E}$, we associate the linear functional $f : \mu \to \mu f$ such that :

$$\mu f = \int_E \mu(dx)f(x),$$

For $\mu \in m\mathfrak{E}$ and $f \in f\mathfrak{E}$, $f \circ \mu$ is the transition kernel having the form :

$$f(x)\mu(A), x \in E, A \in \mathfrak{E}.$$

Definition 1. We say that the Markov chain X verifying $\|P\| < \infty$ is strongly v−stable, if every stochastic kernel Q in the neighborhood $\{Q : \|Q - P\|_v < \epsilon\}$ admits a unique stationary measure ν and :

$$\|\nu - \pi\|_v \longrightarrow 0 \text{ when } \|Q - P\|_v \longrightarrow 0.$$

Theorem 1. *The Harris recurrent Markov chain X verifying $\|P\| < \infty$ is strongly v-stable, if the following conditions are satisfied :*

1. $\exists \alpha \in \mathfrak{M}^+, \exists h \in f\mathfrak{E}^+$ *such that :* $\pi h > 0$, $\alpha \mathbf{1} = 1$, $\alpha h > 0$,
2. $T = P - h \circ \alpha$ *is a nonnegative kernel,*
3. $\exists\ \rho < 1$ *such that,* $Tv(x) \le \rho\, v(x), \forall x \in E$.

where $\mathbf{1}$ *is the function identically equal to 1.*

3. Strong v−stability of the (R, s, S) model

Define on E the σ−algebra $\mathfrak{E}$ engendered by the set of all singletons $\{j\}$, $j \in E$. Consider the function $v(k) = \beta^k$, $\beta > 1$ and define the norm

$$\|\mu\|_v = \sum_{j \in E} v(j)|\mu|(\{j\}), \forall \mu \in m\mathfrak{E}$$

Consider the measure

$$\alpha(\{j\}) = \alpha_j = P_{0j}$$

and the measurable function

$$h(i) = \begin{cases} 1 & \text{if } 0 \le i \le s, \\ 0 & \text{if } s < i \le S. \end{cases}$$

Lemma 1. *X is a Harris recurrent Markov chain.*

Proof. From the definition of the (R, s, S) policy it follows that

$$X_{n+1} = \begin{cases} (S - \xi_{n+1})^+ & \text{if } X_n \le s, \\ (X_n - \xi_{n+1})^+ & \text{if } X_n > s. \end{cases}$$

where $(A)^+ = \max(A, 0)$. Observe that X_{n+1} depends only on X_n and ξ_{n+1}, where ξ_n, $n \ge 1$ are independent and identically distributed random variables and independent from n. Thus, X is a homogenous Markov chain.

The transition probabilities of the chain X from state i to state j are given by

$$P_{ij} = \begin{cases} \sum_{k=S}^{\infty} a_k & \text{if } 0 \leq i \leq s \text{ and } j = 0, \\ a_{S-j} & \text{if } 0 \leq i \leq s \text{ and } 1 \leq j \leq S, \\ \sum_{k=i}^{\infty} a_k & \text{if } s+1 \leq i \leq S \text{ and } j = 0, \\ a_{i-j} & \text{if } s+1 \leq i \leq S \text{ and } 1 \leq j \leq i, \\ 0 & \text{if } s+1 \leq i \leq S \text{ and } j \geq i+1. \end{cases}$$

Thus, X is irreducible and aperiodic. Then, it is Harris recurrent.

Now, we show this result

Lemma 2. *The norm of the transition operator of the chain X is finite, i.e.,*

$$\|P\|_v < \infty$$

Proof. Let's compute $\|P\|_v$. We have

$$\|P\|_v = \sup_{k\in\{0,1,..,S\}} \frac{1}{\beta^k} \sum_{j=0}^{S} P_{kj}\beta^j = \sup(A, B)$$

where

$$A = \sup_{k\in\{0,1,..,s\}} \frac{1}{\beta^k} \sum_{j=0}^{S} P_{kj}\beta^j = \sup_{k\in\{0,1,..,s\}} \frac{1}{\beta^k} \left(\sum_{i=S}^{\infty} a_i + \sum_{j=1}^{S} a_{S-j}\beta^j \right)$$

$$= \sum_{i=S}^{\infty} a_i + \sum_{j=1}^{S} a_{S-j}\beta^j = 1 - \sum_{i=0}^{S-1} a_i + \sum_{i=0}^{S-1} a_i\beta^{S-i} = 1 + \sum_{i=0}^{S-1} a_i \left(\beta^{S-i} - 1\right)$$

and

$$B = \sup_{k\in\{s+1,..,S\}} \frac{1}{\beta^k} \sum_{j=0}^{S} P_{kj}\beta^j = \sup_{k\in\{s+1,..,S\}} \frac{1}{\beta^k} \left(\sum_{i=k}^{\infty} a_i + \sum_{j=1}^{k} a_{k-j}\beta^j \right)$$

$$= \sup_{k\in\{s+1,..,S\}} \frac{1}{\beta^k} \left(1 + \sum_{i=0}^{k-1} a_i \left(\beta^{k-i} - 1\right) \right) < \frac{1}{\beta^{s+1}} \left(1 + \sum_{i=0}^{S-1} a_i \left(\beta^{S-i} - 1\right) \right)$$

$$< 1 + \sum_{i=0}^{S-1} a_i \left(\beta^{S-i} - 1\right) = A$$

So, we have

$$\|P\|_v = 1 + \sum_{i=0}^{S-1} a_i \left(\beta^{S-i} - 1\right) < 1 + \sum_{i=0}^{S-1} a_i \left(\beta^{S} - 1\right) < 1 + \left(\beta^S - 1\right) < \beta^S < \infty$$

Lemma 3. *Let π the stationary distribution of the chain X. Then*

$$\pi h = \frac{1}{1 + H(S-s)} > 0$$

where $H = \sum_{n=1}^{\infty} F_\xi^{*n}$ is the renewal function associated to the cumulative function F_ξ of the random variable ξ_1.

Proof. Consider the Markov chain V given by

$$V_n = S - X_n$$

and denote by X_∞ and V_∞ the random variables distributed with the stationary distributions of the Markov chains X and V respectively. From the definition of the (R, s, S)

policy, we have

$$V_{n+1} = \begin{cases} \min(S, \xi_{n+1}), & \text{if } V_n \geq S - s; \\ \min(S, V_n + \xi_{n+1}), & \text{if } V_n < S - s. \end{cases}$$

If we denote by F_V the distribution function of the random variable V_∞ and we take it as initial distribution for the chain V, then we obtain

$$F_V(x) = P(V_{n+1} < v)$$

$$= P(\min(S, \xi_{n+1}) < v)P(V_n \geq S-s) + P(\min(S, V_n+\xi_{n+1}) < v|V_n < S-s)P(V_n < S-s)$$

For $v \leq S - s$, we obtain

$$F_V(v) = P(\xi_{n+1} < v)P(V_n \geq S - s) + P(V_n + \xi_{n+1} < v|V_n < S - s)P(V_n < S - s)$$

$$F_V(v) = P(\xi_{n+1} < v)P(V_n \geq S - s) + \sum_{j=0}^{v-1} P(V_n = j)P(\xi_{n+1} < v - j)$$

$$= P(\xi_{n+1} < v)P(V_n \geq S - s) + (F_V * F_\xi)(v)$$

So, for all $v \leq S - s$

$$F_V(v) = CF_\xi(v) + (F_V * F_\xi)(v)$$

or

$$F_V = CF_\xi + F_V * F_\xi$$

witch is a renewal type equation, where, $C = P(V_n \geq S - s) = 1 - F_V(S - s)$.

Substitute F_V in the right side of the last equation

$$F_V = CF_\xi + (CF_\xi + F_V * F_\xi) * F_\xi$$

Thus,

$$F_V = CF_\xi + CF_\xi^{*2} + F_V * F_\xi^{*2}$$

Substitute F_V again and again

$$F_V = CF_\xi + CF_\xi^{*2} + CF_\xi^{*3} + F_V * F_\xi^{*3}$$

$$\vdots$$

$$F_V = CF_\xi + CF_\xi^{*2} + CF_\xi^{*3} + \ldots + CF_\xi^{*n} + F_V * F_\xi^{*n}$$

$$F_V = C\sum_{i=1}^{n} F_\xi^{*i} + F_V * F_\xi^{*n}$$

$\sum F_\xi^{*i}$ converges because F_V exists (and $0 \leq F_V \leq 1$). This implies that $F_\xi^{*n} \to 0$ as $n \to \infty$. Thus, $F_V * F_\xi^{*n} \to 0$ as $n \to \infty$ and we obtain

$$F_V(v) = C\sum_{i=1}^{\infty} F_\xi^{*i}(v) = CH(v), \quad \text{for } v \leq S - s.$$

Now, we aim to find the value of C. We have

$$C = 1 - F_V(S - s) = 1 - CH(S - s) \Rightarrow C(1 + H(S - s)) = 1 \Rightarrow C = \frac{1}{1 + H(S - s)}.$$

Thus,

$$F_V(v) = \frac{H(v)}{1 + H(S - s)}, \quad \text{for } v \leq S - s.$$

Now, compute πh. We have

$$\pi h = \sum_{i=0}^{S} \pi_i h(i) = \sum_{i=0}^{s} \pi_i = 1 - \sum_{i=s+1}^{S} \pi_i = 1 - P(X_\infty > s)$$

$$= 1 - P(S - V_\infty > s) = 1 - P(V_\infty < S - s) = 1 - F_V(S - s) = 1 - \frac{H(S - s)}{1 + H(S - s)}.$$

So,

$$\pi h = \frac{1}{1 + H(S-s)} > 0.$$

Lemma 4. *Denote by* $\mathbf{1}$ *the function identically equal to 1. Then*

$$\alpha\mathbf{1} = 1 \quad \textit{and} \quad \alpha h > 0$$

Also, the operator $T = P - h o \alpha$ *is nonnegative.*

Proof. We have

$$\alpha\mathbf{1} = \sum_{j=0}^{S} P_{0j} = \sum_{i=S}^{\infty} a_i + \sum_{j=1}^{S} a_{S-j} = \sum_{i=S}^{\infty} a_i + \sum_{j=0}^{S-1} a_j = \sum_{i=0}^{\infty} a_i = 1$$

Now, compute αh

$$\alpha h = \sum_{j=0}^{S} \alpha(\{j\}) h(j) = \sum_{j=0}^{S} P_{0j} h(j) = \sum_{j=0}^{s} P_{0j}$$

$$= \sum_{i=S}^{\infty} a_i + \sum_{j=1}^{s} a_{S-j} = \sum_{i=S}^{\infty} a_i + \sum_{i=S-s}^{S-1} a_i = \sum_{i=S-s}^{\infty} a_i > 0$$

Also, we can easily verify that

$$T(i, \{j\}) = T_{ij} = P_{ij} - h(i)\alpha_j = \begin{cases} 0 & \text{If } 0 \le i \le s, \\ P_{ij} & \text{Otherwise.} \end{cases}$$

so, T is a nonnegative kernel.

Lemma 5. *Let*

$$\rho = \frac{\sum_{i=s+1}^{\infty} a_i}{\beta^{s+1}} + \sum_{i=0}^{s} a_i \beta^{-i}$$

Then $Tv(k) \le \rho v(k)$ *for all* $k \in E$ *and* $\rho < 1$, $\forall \beta > 1$.

Proof. Compute $Tv(k)$:

$$Tv(k) = \sum_{j=0}^{S} T_{kj} v(j)$$

If $0 \le k \le s$ then :

$$Tv(k) = 0$$

If $s < k \le S$, then we have

$$Tv(k) = \sum_{j=0}^{k} P_{kj}\beta^j = \sum_{i=k}^{\infty} a_i + \sum_{j=1}^{k} a_{k-j}\beta^j = \sum_{i=k}^{\infty} a_i + \sum_{i=0}^{k-1} a_i \beta^{k-i}$$

$$= \sum_{i=k}^{\infty} a_i + \beta^k \sum_{i=0}^{k-1} a_i \beta^{-i} = \left(\frac{\sum_{i=k}^{\infty} a_i}{\beta^k} + \sum_{i=s+1}^{k-1} a_i \beta^{-i} + \sum_{i=0}^{s} a_i \beta^{-i} \right) \beta^k$$

$$\le \left(\frac{\sum_{i=k}^{\infty} a_i}{\beta^{s+1}} + \frac{\sum_{i=s+1}^{k-1} a_i}{\beta^{s+1}} + \sum_{i=0}^{s} a_i \beta^{-i} \right) \beta^k \le \left(\frac{\sum_{i=s+1}^{\infty} a_i}{\beta^{s+1}} + \sum_{i=0}^{s} a_i \beta^{-i} \right) \beta^k = \rho v(k)$$

Also, for $\beta > 1$

$$\rho = \frac{\sum_{i=s+1}^{\infty} a_i}{\beta^{s+1}} + \sum_{i=0}^{s} a_i \beta^{-i} < \sum_{i=s+1}^{\infty} a_i + \sum_{i=0}^{s} a_i = 1$$

Then, we can show this result

Theorem 2. *In the (R, s, S) inventory model with zero leadtimes, the Markov chain $X = (X_n)_{n\geq 1}$ representing the inventory level at end of periods is strongly $v-$stable with respect to a function $v(k) = \beta^k$ for all $\beta > 1$.*

proof. To prove this result, we apply the operator's approach [1]. Lemmas (1) and (2) allow us to use theorem (1). All conditions for the strong stability (theorem (1)) are satisfied and are given by the above results (lemmas (3),(4) and (5)).

This result means that a small perturbation of the demand probabilities generates only a small deviation of the stationary distribution of embedded Markov chain X and therefore, a small deviation of the characteristics of the system depending on this distribution. This fact allows us to use approximations on the demand probabilities.

The (R, S) model is a special case of the (R, s, S) one. It is widely used in practice. From the above theorem, we can deduce this result

Corollary 1. *In the (R, S) inventory model with zero leadtimes, the Markov chain $Y = (Y_n)_{n\geq 1}$ representing the inventory level at end of periods is strongly $v-$stable with respect to a function $v(k) = \beta^k$ for all $\beta > 1$.*

Proof. It suffice to take $s = S - 1$ in the above proofs.

Conclusion

We have proved the strong $v-$stability of the embedded Markov chain in the considered inventory model. This allow us to justify approximations used in practice to analyse such inventory systems. Furthermore, strong stability method can be used to obtain quantitative estimates of the approximation error [8] and to measure its performance, we can build an algorithm as in [3]. Also, other inventory models can be studied and the result should be extended.

Bibliography

1. D. Aïssani and N.V. Kartashov, *Ergodicity and stability of Markov chains with respect to operator topology in the space of transition kernels*, Dokl. Akad. Nauk. Ukr. SSR, ser. A **11** (1983), 3-5.
2. K.J. Arrow, T. Harris and J. Marschak, *Optimal Inventory Policy*, Econometrica **19** (1951), 250-272.
3. L. Boualouche and D.Aïssani, *Performance Evaluation of an SW Communication Protocole (Send and Wait)*, Proceeding of the MCQT'02 (First Madrid Conference on Queuing Theory), Madrid (Spain) (2002), 18.
4. E.S. Boylan, *Stability theorems for solutions to the optimal inventory equation*, Journal of Applied Probability **6** (1969), 211–217.
5. F. Chen and Y.S. Zheng, *Sensitivity analysis of an (s, S) inventory model*, Operations Research Letters **21(1)** (1997), 19–23.
6. F.W. Harris, *How many parts to make at once*, Factory (The magazine of management) **10(2)** (1913), 135–136.
7. S. Karlin, *Dynamic inventory policy with varing stochastic demands*, Management Sciences **6** (1960), 231–258.
8. N.V. Kartashov., *Strong Stability of Markov Chains*, VNISSI, Vsesayouzni Seminar on Stability Problems for Stochastic Models, Moscow (1981), 54-59; English transl. in J. Soviet Mat. **34** (1986), 1493-1498.
9. N.V. Kartashov, *Strong Stable Markov Chains*, VSP, Utrecht, TbiMC Scientific Publishers, 1996.
10. I. Sahin., *Regenerative inventory systems*, Wiley, New York, 1989.
11. A. Veinott, *Optimal policy in a dynamic, single product, non-stationary inventory model with several demand classes*, Operations Research **13** (1965), 761-778.

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vol. 10(26) **THEORY OF STOCHASTIC PROCESSES** issue 3–4
2004

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Журнал зареєстрований 31 грудня 2003 року. Реєстраційний номер КВ8291. Засновники: Інститут математики НАНУ, Інститут прикладної математики та механіки НАНУ, мале підприємство «TBiMC».

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Надруковано в МП «TBiMC»: 03127, Київ, пр. Академіка Глушкова, 2.

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