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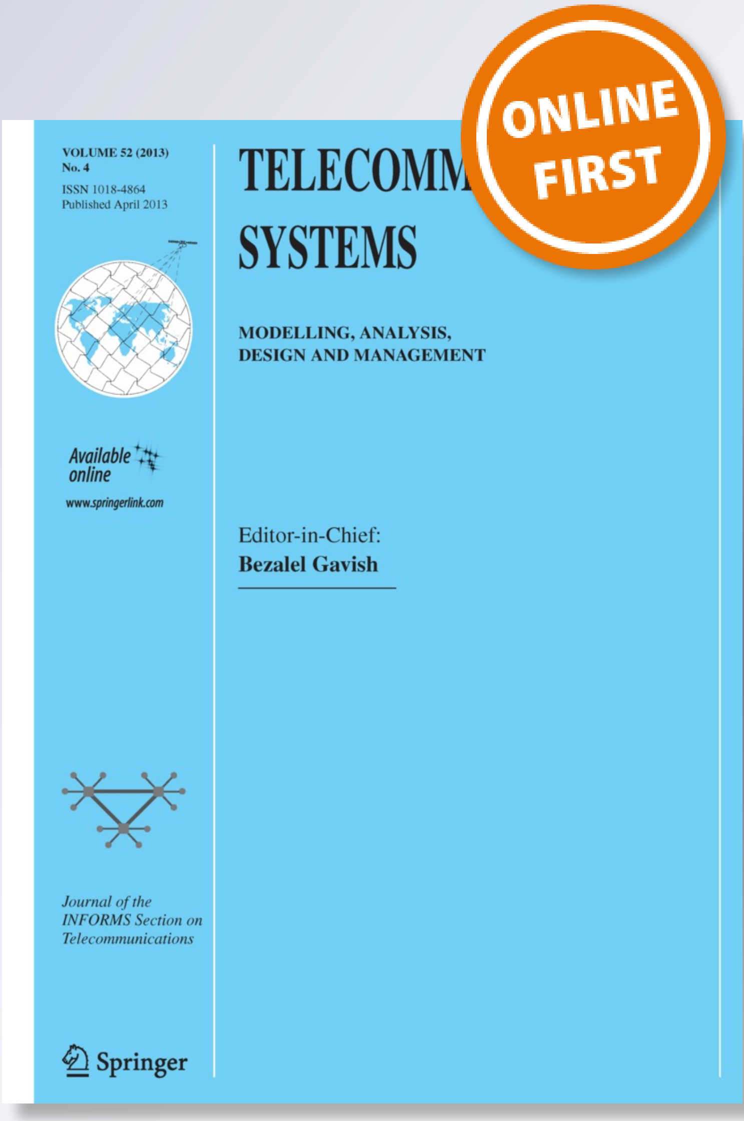

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A stochastic model to study the impact of the transmission frequency of hello messages on the connectivity of ad hoc networks

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Abstract In this paper, we exploit the utility of Hello messages in the Ad hoc networks to study the impact of their transmission frequencies on the connectivity of the network. Assuming that the Hello messages arrive at a given node according to a Poisson process, we model a cluster-head motion (respectively an ordinary node motion) using a random process. This model allows us to find the critical value of the transmission frequency of Hello messages. We also have investigate a fundamental property of an Ad hoc network: its connectivity. We then analyze the number of neighbors of a given node, the isolation probability, the handoff probability and the probability that the considered network is connected, i.e. each node can communicate with an other node via the network.

Keywords Ad hoc networks · Hello message management · Transmission frequency · Random process · Network connectivity

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1 Introduction

The wireless Ad hoc networks MANET (Mobile Ad hoc NETworks) emerged as a category of wireless networks which use the radio interface and are self-organizing and self-administering [3, 20] without a fixed infrastructure. A MANET is a collection of autonomous mobile nodes communicating via radio links. We consider an Ad hoc network, which is managed in a set of groups called clusters. Each cluster consists of a number of nodes; one of them is the cluster-head, responsible for the resource allocation to all nodes belonging to its cluster. The set of cluster-heads constitutes the *dominant set* [32]. The utility of the infrastructures, in the wireless networks which are equipped with infrastructures, is replaced in the Ad hoc networks by control messages called "Hello messages". The purpose of using of Hello messages is the determination of paths when one assumes that these messages indicate a reliable communication with the source.

The basic protocol proposing the use of Hello messages to maintain connectivity was initially described in OSPF (Open Shortest Path First) [21]. The nodes regularly transmit Hello messages to indicate their presence to their neighborhood. The frequency of these messages is noted f_{hello} and the delay between two successive messages is d_{hello} (equal to $1/f_{hello}$). However, these messages occupy a good part of the bandwidth and sometimes can cause an overflow. Sending these messages at low frequency causes a considerable loss of information and using a very high frequency saturates the bandwidth on the detriment of useful data. In [6], the authors proved that the effectiveness of the protocols using the Hello messages depended on the size of these messages, the transmission frequency and the lifetime of an entry in the routing table. Although these criteria are very important, little research is directed in this way.

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In this work, we consider the problem of the transmission management of Hello messages to answer some questions: can we space out the sending of Hello messages? What is the critical value of the transmission frequency? What impact can this frequency have on the network connectivity?

Under the assumption that these messages arrive according to a Poisson process with rate λ (in Sect. 3, we discuss the validity of this assumption), we calculate the probability that a link is broken knowing that it was previously available. Indeed, the fragility of the use of link radio encourages us to take full advantage of an available link. We also model a cluster-head motion (respectively an ordinary node motion) by a Markov Process $\{Xn\}$ (respectively $\{Yn\}$) with continuous space state. We establish a relation between the rate λ and the isolation probability of a node, and calculate the connectivity probability as a function of λ. Bettstetter [5] presented a framework for the calculation of stochastic connectivity properties of wireless multihop networks and obtained interesting results when the node distribution was uniform in the analyzed area A. For a detailed survey about the connectivity analysis in Ad hoc networks, we refer to [19]. The approach we use here is interesting because the results are obtained according to the transmission frequency of the Hello messages and the information contained in these messages.

In [12, 13, 31] and [16], it was explained that since the connectivity of an Ad hoc network depended on the network characteristics (ex: density, speed, nodes position . . .) which were sometimes variable, there was a need to create adaptive protocols for the transmission management of Hello messages.

In [16], the authors proposed a new adaptive protocol called "A Turnover based Adaptive Hello Protocol for Mobile Ad hoc and Sensor Networks (TAP)". They introduced a new metric (r) called sales turnover. During one period $\triangle t$, r corresponded to the ratio of the number of guaranteed neighbors at $t + \triangle t$ and the number of neighbors at the moment t. After each calculation of r, two cases could arise:

- If $r < r_{opt}$, this meant that f_{hello} was high, and there are no appreciable changes in the routing table. The frequency f_{hello} had therefore to be reduced.
- If $r > r_{opt}$, f_{hello} was low and there were many changes in the network. f_{hello} had therefore to be increased.

The major disadvantage of this protocol is the lack of a method for the calculation of r_{opt} and the value of $\triangle t$.

Others works dealt with the study of the transmission frequency value and the signal strength. Nayebi and Sarbazi-Azad [22] studied the problem of determining the maximum "hello" interval preserving the connectivity of homogeneous topology with high probability. In [4], authors proposed to use signal strength to determine if the link quality was improving or deteriorating.

In the analytical studies for connectivity analysis of wireless Ad hoc networks, closed form expressions for the spatial node distribution are very important to understand the behavior of the network. In the majority of studies done so far, authors have considered that the station locations were assumed to be uniformly distributed in a region. This assumption however might not be valid in practice.

In this paper, we first obtain the spatial nodes distribution using the information found on the Hello messages. Then, using this distribution and the Hello messages frequency, we determine close form expressions for different connectivity proprieties.

The rest of the paper is organized as follows: In Sect. 2, we present some definitions concerning the multi-clusters Ad hoc networks. In Sect. 3, we give a simulation study about the impact of the transmission frequency of Hello messages on the bandwidth congestion. Section 4 is devoted to the problem setting and modelling. In Sect. 5, we present the analysis of the distance between a node and its cluster-head which allows us to obtain a relation between the disconnection probability and the Hello messages transmission frequency. In Sect. 6, we determine the transition probabilities. Section 7 summarizes various results concerning network connectivity. Validation of our model with simulation is given in Sect. 8. The last section concludes and presents some suggestions for further investigations in this area.

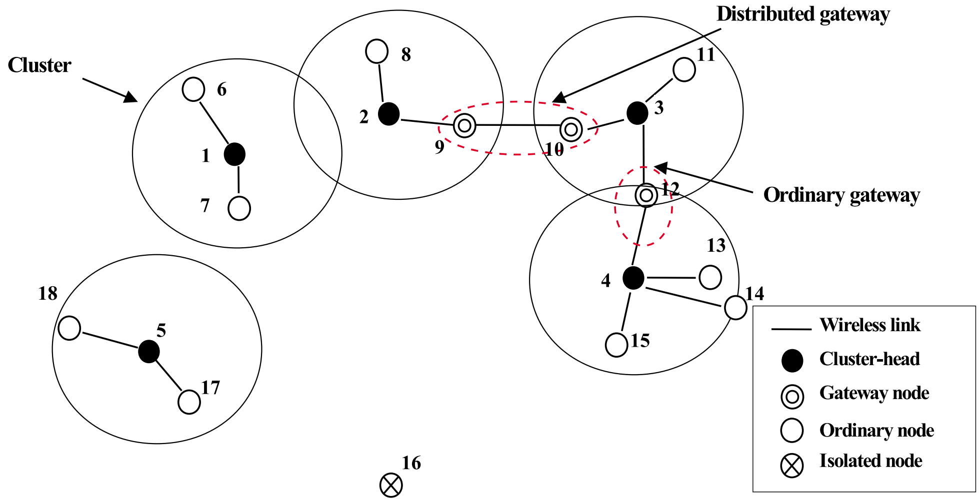
2 Network model

We consider a mobile Ad hoc network managed within a set of clusters. Clustering was found to be an effective means of resource management for MANETs regarding network performance, routing protocol design, Quality of Service (QoS) and network modeling [29].

Each cluster consists of a number of nodes which are classified into two types:

Cluster-head nodes: for any efficient cluster operation (cluster being a subset of nodes in a network satisfying a particular property), there must be a support or backbone to sustain all essential control functions such as channel access, routing, calculation of the routes for longer-distance messages, bandwidth allocation, forwarding inter-cluster packets, power control and virtual-circuit support [24]. Two cluster-heads are linked (connected) either directly or via gateway nodes, and they will have the subordinate nodes of that cluster linked to them. In Fig. 1, cluster-heads 2 and 3 are connected, but there are no link between the two cluster-heads 1 and 5. To send a packet, an ordinary node must first direct this packet to its cluster-head. Should the receiver share the same cluster location, the cluster-head will then send it the packet. However, if the receiver is in a different cluster location, the cluster-head will route this packet to another cluster-head directly connected to the receiver, and the

Fig. 1 Multi-clusters Ad hoc network



last cluster-head will forward this packet to its final destination [8].

Ordinary Nodes: which are two kinds:

Gateway Node: This is a node that works as the common or distributed access point for two cluster-heads. When a node remains within the transmission range of two cluster-heads, like node 12 in Fig. 1, it is called an ordinary gateway for two corresponding clusters. On the other hand, a node having one cluster-head as an immediate neighbor, and which can in addition reach a second cluster-head in two hops, like nodes 9 or 10, is a distributed gateway that is linked to another distributed gateway of another cluster. Both distributed gateways provide the path for the inter-cluster communication [25].

Simple Node: Simple nodes do not perform any other function besides a normal node role. They are members of an exclusive cluster independent of the neighbors belonging to a different cluster.

3 Effect of the frequency of Hello messages on the available bandwidth

The available bandwidth decreases as the frequency of Hello messages increases [13].

In this section, we show, using simulations, the impact of the transmission frequency of Hello messages on the bandwidth performance.

For different value of the transmission frequency, and in the presence of data messages, we determine the mean sojourn time (see Figs. 2 and 3).

This scenario is repeated for these different situations:

- data messages frequency is low,
- data messages frequency is average,
- data messages frequency is high.

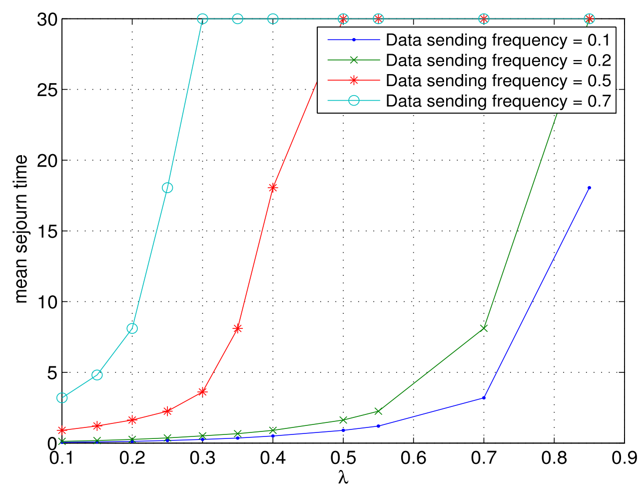


Fig. 2 Bandwidth congestion

4 Problem setting and modeling

The cluster-head (ordinary node) is mobile (random trajectories), and periodically sends information about its position and its speed in Hello messages. All nodes continuously manage the signal strength of these messages. If the distance between an ordinary node and its cluster-head is less than the transmission range, the ordinary node stays in its cluster. Otherwise, it tries to handover to another cluster-head.

Let us now assume that the position of the cluster-head and of the ordinary node at the time of reception of the n-th Hello message are known [23]. Could we obtain any information on their positions before the $(n+1)$-th Hello message is received? Knowing these forecasts allows us to predict that this ordinary node will remain in the transmission range of its cluster-head. In this case, we will take some precautions to avoid the rupture of the link.

We denote by $X_n = (x_n, y_n)$ (respectively $Y_n = (x'_n, y'_n)$) the position of a cluster-head (respectively an ordinary node)

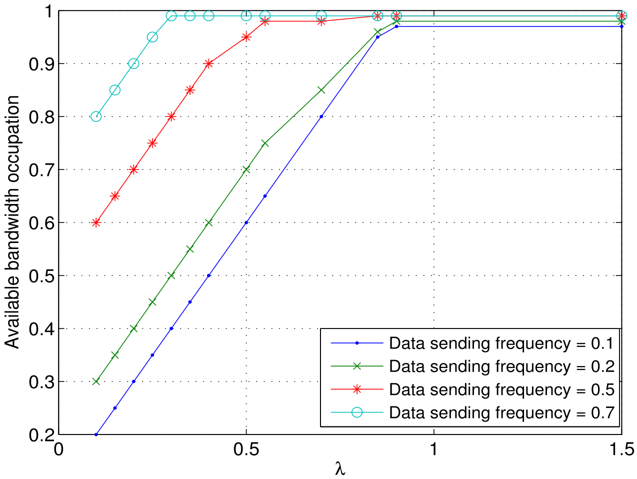


Fig. 3 Bandwidth occupation

at the time when the n-th Hello message arrives, and t_n be the instant of reception of n-th Hello message. The following events can occur in the time interval $(t_n, t_n + \triangle t)$:

- the Hello message arrives at the considered node with a probability equal to $\lambda\triangle t + \mathcal{O}(\triangle t)$, as suggested in Gelenbe [10];
- Said otherwise, the probability that the Hello message does not arrive (because of a link breaking, a transmission failure or an error) is $1 - (\lambda\triangle t + \mathcal{O}(\triangle t))$.

Assuming here that the current reception is independent from the previous ones, then the arrival times of Hello messages form a Poisson process with rate $\lambda > 0$. This assumption is natural in our case because the moment of arrival of the Hello message is random, and the probability of receiving at the same moment two Hello messages is practically negligible. This characterizes a Poisson process; the reader can find a description of this process and its proprieties in [17, 26].

Consequently, the time durations between X_{n+1} and X_n as well as between Y_{n+1} and Y_n are mutually independent and exponentially distributed. At the instant when the $(n+1)$-th Hello message arrives, the position of the cluster-head (ordinary node) depends only on its position at the instant when n-th Hello message arrived and not on its previous positions. Therefore, the Process $\{X_n\}$ (respectively $\{Y_n\}$) is a Markov one with continuous state space and discrete time.

Let W_n be the inter-arrival time between the $(n+1)$-th and n-th Hello messages (it follows an exponential law with rate λ), $V^h > 0$ (respectively $V^e > 0$) be the speed of the cluster-head (respectively ordinary node), and θ_n^h (respectively θ_n^e) be the angle determined by the direction of the cluster-head (respectively ordinary node) and the $(0X)$ axis. So the future locations can be determined from

$$x_{n+1} = x_n + V^h W_n \cos\theta_n^h,\ y_{n+1} = y_n + V^h W_n \sin\theta_n^h,$$

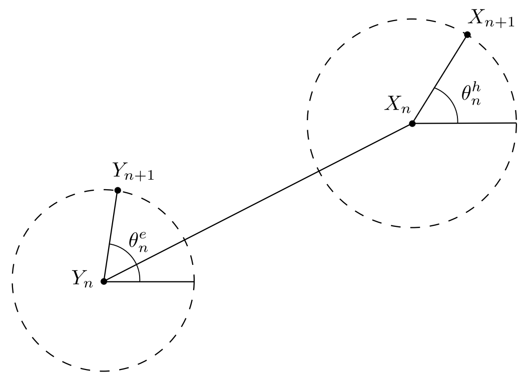


Fig. 4 Node positions

and

$$x'_{n+1} = x'_n + V^e W_n \cos\theta_n^e,\ y'_{n+1} = y'_n + V^e W_n \sin\theta_n^e.$$

Figure 4 shows the positions of the cluster-head and the ordinary node when the $(n+1)$-th Hello message is received.

5 Analysis of the distance between a node and its cluster-head

In [15], authors gave an approach to predict the time dependence of link connectivity using a Markov model. In this section, we calculate the probability that a connected link remains connected, and obtain an expression of a connected link using information found in Hello message and the transmission frequency of these messages.

Let $d(X_{n+1}, Y_{n+1})$ be the distance between an ordinary node located at X_{n+1} and its cluster-head located at location Y_{n+1}. We want to calculate

$$P\big(d(X_{n+1}, Y_{n+1}) > R\big),$$

which represents the disconnection probability. If we note

$$D_{n+1} = d(X_{n+1}, Y_{n+1}),$$

then the worst situation on can have is that the two nodes go in opposite directions (see Fig. 4), and we have

$$\begin{aligned} D_{n+1} &\le d(X_n, Y_n) + \text{the first circle range} \\ &\quad + \text{the second circle range} \\ &= d(X_n, Y_n) + V^h W_n + V^e W_n. \end{aligned}$$

Thus

$$P(D_{n+1} > R) \ge P\left(W_n > \frac{R - d(X_n, Y_n)}{V^h + V^e}\right).$$

If we define α as

$$\alpha = \frac{R - d(X_n, Y_n)}{V^h + V^e},$$

then we have

$$P(D_{n+1} > R) \geq P(W_n > \alpha) = \exp(-\alpha\lambda). \tag{1}$$

The parameter λ is the transmission rate; it represents the mean number of received messages per unit time. So, the mean time between two Hello messages is $\frac{1}{\lambda}$. We want to find the value of λ that makes the probability (1) as small as possible. Since this probability depends on λ and α (hence R), we have to take in consideration the batteries of the nodes. Consequently, the value of λ can be calculated by solving Eq. (2)

$$P(D_{n+1} > R) \leq \epsilon. \tag{2}$$

The critical value of the transmission range R was found in [28]. It depend on the number of nodes. Using that results in (2) allows us to calculate λ.

To get an idea of the connectivity between an ordinary node and its cluster-head, we perform a simulation study. We place a node and its cluster-head in arbitrary positions and we check whether the link becomes unavailable. For this purpose, we have programmed a tool in NS-2. The same experiment is repeated Ω times to compute the percentage of connected links, which is simply

$$\frac{\text{number of connected links}}{\Omega}.$$

For Ω sufficiently large, we obtain a good estimate for $P(D_{n+1} > R)$.

Figure 5 shows the simulation results and analytical ones for $P(D_{n+1} > R)$. We see there that the difference between the two plots (theoretical and simulation results) is getting smaller when the previous distance between the ordinary node and its cluster-head is increasing. In this case, the disconnection probability approaches the limit given by formula (1). We conclude that if two nodes are already far from each other at the moment of reception of the n-th Hello message, then the probability that the link breaks at the moment of reception of the $(n+1)$-th Hello message increases.

The plots in Fig. 5 clearly show that the probability of disconnection that we found by simulation is larger than $\exp(-\alpha\lambda)$ for all values of α or any value of $d < R$, (d meaning here $d(X_n, Y_n)$). These results can therefore be interpreted in a sens as a confirmation of the correctness of our study.

The above presentation of the relation between the disconnection probability, and λ and R, is useful in practice for topology design of Ad hoc networks and increases the utility of Hello messages by determining the critical value of the time duration between two sending of these messages.

6 Transition probabilities

Let $X = (x, y)$ and $Y = (x_1, y_1)$ be two points of the plan. The transition probability

$$P(X_{n+1} = X / X_n = Y)$$

can be defined as

$$\begin{aligned} &P(X_{n+1} = X / X_n = Y) \\ &\quad = P\big(x = x_1 + w_n \cos\theta_n^h V^h, y = y_1 + w_n \sin\theta_n^h V^h\big) \\ &\quad = \frac{1}{2\pi\sqrt{(x-x_1)^2 + (y-y_1)^2}} \\ &\qquad \times P\left(w_n = \frac{\sqrt{(x-x_1)^2 + (y-y_1)^2}}{V^h}\right) \\ &\quad = \frac{1}{2\pi\sqrt{(x-x_1)^2 + (y-y_1)^2}}\lambda \\ &\qquad \times \exp\left\{-\lambda\frac{\sqrt{(x-x_1)^2 + (y-y_1)^2}}{V^h}\right\}. \end{aligned} \tag{3}$$

In the same way, the conditional probability $P(X_{n+1} = X / X_n = X)$ is determined.

In fact,

$$\begin{aligned} &P(X_{n+1} = X / X_n = X) \\ &\quad = P\big(x = x + w_n \cos\theta_n^h V^h, \\ &\qquad\quad y = y + w_n \sin\theta_n^h V^h / X_n = X\big) \\ &\quad = P\big(w_n \cos\theta_n^h V^h = 0, w_n \sin\theta_n^h V^h = 0 / X_n = X\big). \end{aligned}$$

Under the assumption that $V^h \neq 0$ and since $P(X_{n+1} = X / X_n = X)$, then $\theta_n^h = 0$. Hence

$$P(X_{n+1} = X / X_n = X) = P(w_n = 0) = 0. \tag{4}$$

This is because

$$P(w_n = t) = \lambda e^{-\lambda t} t > 0$$

and $P(w_n = 0) = P(\text{that the } n\text{-th and the} (n+1)\text{-th Hello}$ messages arrive at the same time).

However, since Hello messages arrive according to a Poisson process, then $P(w_n = 0) = 0$. It is a law of probability, meaning we can easily show that

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P\big(X_{n+1} = (x, y) / X_n = (x_1, y_1)\big)\, dx_1\, dy_1 = 1.$$

In the same way, we can calculate $P(Y_{n+1} = X' / Y_n = Y')$.

If $X' = (x', y')$ and $Y' = (x'_1, y'_1)$ are two points of the plan, then

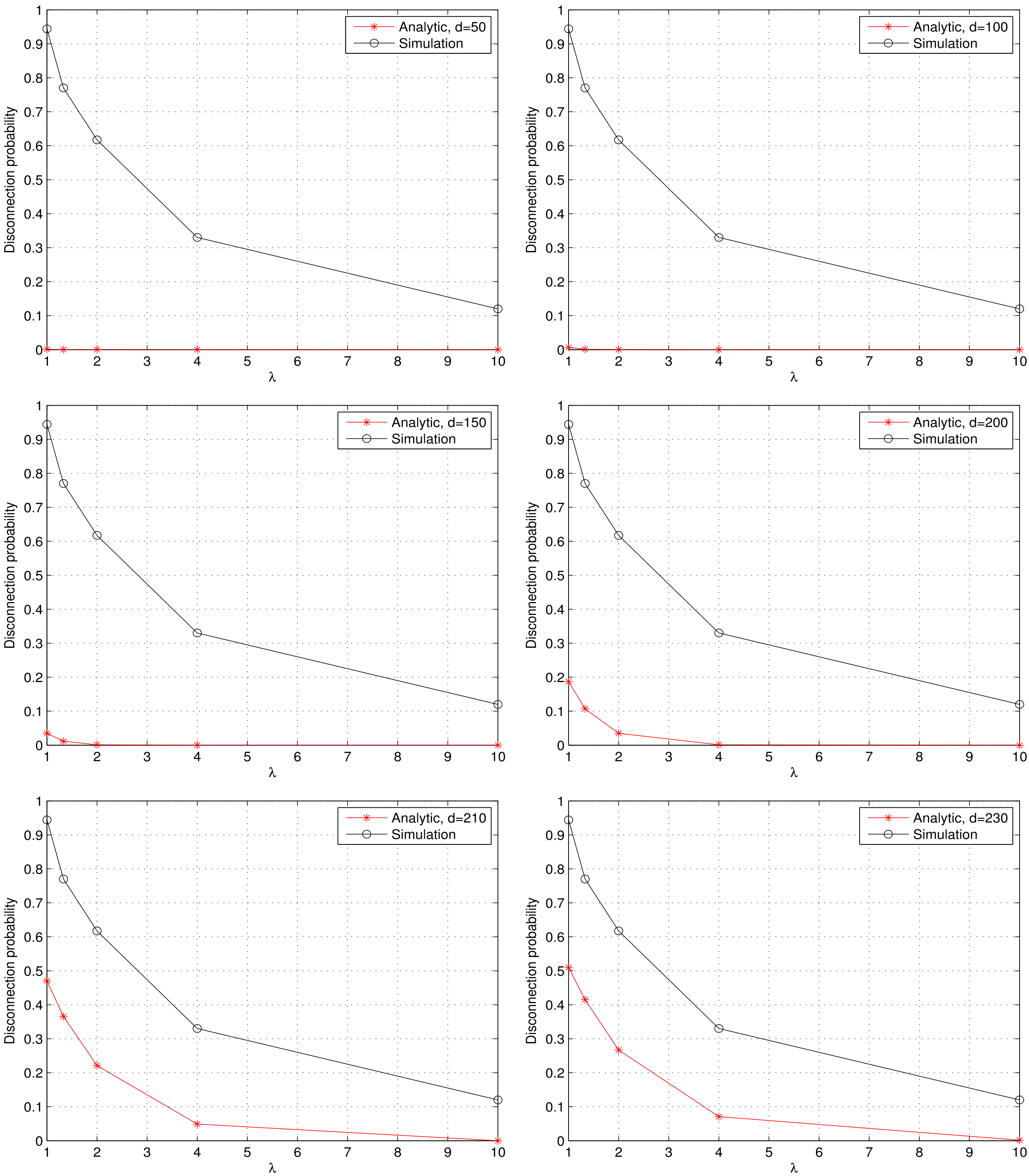


Fig. 5 Variation of disconnection probability according to λ and *d*

$$P\left(Y_{n+1} = X'/Y_n = Y'\right) = \frac{1}{2\pi\sqrt{(x'-x_1')^2+(y'-y_1')^2}}\lambda \times \exp\left\{-\lambda\frac{\sqrt{(x'-x_1')^2+(y'-y_1')^2}}{V^e}\right\}. \quad (5)$$

Before we go on, let us remind the reader of some results found in the literature. The authors of [18] considered the planar motion of a particle that was moving with constant finite speed c, and changing direction θ according to Uniform law in $]0, 2\pi)$, and obtained the explicit conditional distribution of the probability law $f(x, y, t)$ of $(X(t), Y(t))$, where $(X(t), Y(t))$ was the particle's position at $t \geq 0$. It was also demonstrated in [9] that the probability density $p(x, y, t)$, in the long time limit, depended on the moments of the angular distribution used to determine the direction of movement at each step of the random walk.

In this work, the problem is the same, but in our case, we do now have the information at time n, and we want to get estimates for the node's position at time $n + 1$. We are not trying to obtain the steady state of the process, but instead want to know its possible evolution between the present moment and the near future.

If we choose the origin to be the node position at the time n and then make a change of variable, the Eqs. (3) and (5) become respectively

$$P\left(X = (x, y)\right) = \frac{1}{2\pi\sqrt{x^2+y^2}}\lambda \exp\left\{-\lambda\frac{\sqrt{x^2+y^2}}{V^h}\right\}, \quad (6)$$

and

$$P\left(Y = (x, y)\right) = \frac{1}{2\pi\sqrt{x^2+y^2}}\lambda \exp\left\{-\lambda\frac{\sqrt{x^2+y^2}}{V^e}\right\}. \quad (7)$$

7 Node degree analysis

The node degree is the number of nodes which are in its transmission range. In this section, we first compute the probability that a node (ordinary or cluster-head) has degree k. We also obtain the mean degree of a given node (which is the mean number of neighbors), for a cluster-head, its mean degree represents the average number of nodes under its governorship.

Let us consider a node located at $X = (x, y)$, and let R and V be respectively its transmission range and its speed (we suppose here that all network nodes have the same speed V). A second node is randomly placed according to the probability distribution function $P(X)$. The two nodes will establish a link if the second node is placed in the disc of radius R centered at X [14]. The probability of this link event is:

$$P_0(X) = \int_{y-R}^{y+R}\int_{x-\sqrt{R^2-(y'-y)^2}}^{x+\sqrt{R^2-(y'-y)^2}} P\left(x', y'\right) dx'dy' = 1 - \exp\left\{-\frac{\lambda R}{V}\right\}. \quad (8)$$

In a network with l ordinary nodes and m cluster-heads, the probability that a cluster-head has degree k can be defined as

$$P(D = k/X) = \binom{k}{l+m-1} P_0^k(X)\left(1 - P_0(X)\right)^{l+m-k-1}. \quad (9)$$

Its mean degree is

$$\mathbb{E}(D/X) = (l+m-1)P_0(X) = (l+m-1)\left(1 - \exp\left\{-\frac{\lambda R}{V}\right\}\right). \quad (10)$$

Here $\mathbb{E}(D/X)$ denotes the mean degree of randomly chosen node in the network.

The probability that a given node is isolated is therefore

$$P(D = 0/X) = \left[1 - P_0(X)\right]^{l+m-1} = \left(\exp\left\{-\frac{\lambda R}{V}\right\}\right)^{l+m-1}. \quad (11)$$

This last equation is obtained by simply replacing k by 0 in (9).

In a mobile scenario, if an isolated node wants to send or receive information, it must wait until it gets in the range of another node or until another node passes by. In the case of an isolated cluster-head, this means that the network needs to elect a new dominant set (using a clustering algorithm, like the Weighted Clustering Algorithm (WCA) [7]).

The probability, which is here

$$P_1(X) = \left(\exp\left\{-\frac{\lambda R}{V}\right\}\right)^m, \quad (12)$$

means that the node located at X can not communicate with any cluster-head. Therefore, the dominant set must be updated because it can no longer play its role.

An ordinary node is not covered by its cluster-head if it is not in its transmission range. The probability of this event, which represents the probability of a handoff (in this case, the WCA procedure will assign it to another cluster-head), is given by:

$$P = 1 - P_0(X) = \exp\left\{-\frac{\lambda R}{V}\right\}. \quad (13)$$

This result is obtained from formula (8).

We have so studied the level of connectivity from the viewpoint of a single node and its neighborhood. We now consider the connectivity of all nodes from a global network viewpoint. The property "being connected" is one of the essential characteristics of any network.

A multi-cluster Ad hoc network is connected if and only if there is at least a path between each pair of cluster-heads and all the ordinary nodes are not isolated.

It is very difficult to calculate the probability that the network is connected $P(con)$. However, it can be approximated by the probability of having no isolated nodes:

$P(con) \simeq P(no\ isolated\ node)$.

The probability that this network is connected is given by the following proposition:

Proposition 7.1 *The probability that the network is connected is*

$$P(con) \simeq \left(1 - \left(\exp\left\{-\frac{\lambda R}{V}\right\}\right)^{l+m-1}\right)^{l+m}. \qquad (14)$$

Proof From formula (11), one can deduce that

$$\left(1 - \left(\exp\left\{-\frac{\lambda R}{V}\right\}\right)^{l+m-1}\right)$$

is the probability that a given node is not isolated. Therefore, the probability that the $(m+l)$ nodes are not isolated is

$$P(con) \simeq \left(1 - \left(\exp\left\{-\frac{\lambda R}{V}\right\}\right)^{l+m-1}\right)^{l+m}. \qquad \square$$

If we want to have $P(con) > 0.99$, we have to find the pair (R, λ) which gives this probability. Moreover, for fixed R, we find a relation between the total number of nodes $m + l$ and λ.

The reason why we are interested in the pair (R, λ) which gives an almost surely connected network is the following: if the range R of all nodes is set to a very high value, the network is surely connected. However, this guaranteed connectivity has its price: a higher range requires a higher transmission power and causes more interference between the nodes.

Several studies on the connectivity of mobile Ad hoc networks have been undertaken for the last years. They can be divided into two categories. The first category, when studying connectivity is mainly interested in obtaining various indices (isolation probability, probability of a handover, connection probability, ...). The second deals with the evaluation of the critical transmission range of nodes which will guarantee the connectivity of the Ad hoc networks. The study presented here focuses only on the homogeneous Ad hoc networks, and our work can be seen as an intermediate category where the results we obtain allow us to find the pair (λ, R) required to keep the network connected.

8 Simulation experiments

For our simulation, we used the *Highest-Degree*, or 'connectivity-based clustering'. It is an original proposal [11] in which the degree of a node is calculated on the basis of its relative proximity to other nodes. Each node transmits its information to others within its transmission range. A node u is considered to be a neighbor of another node v if u lies within the transmission range of v. The node having the greatest number of neighbors (i.e., most/highest degree of direct transmission links) is chosen as cluster-head. The neighbors of a cluster-head become absorbed as members of that cluster (or specific neighborhood) and cannot participate any further in the election process now that they have a declared 'home'. The election process thus prevents any direct link between cluster-heads; only one cluster-head will reside in each cluster.

We perform here some simulations in NS-2 to check the asymptotic results presented in the previous section. We place $l + m$ nodes in a large area and take as input the parameters of the Random Waypoint Model by assigning the same speed to every node in the simulation and a zero pause time.

We note that the movement pattern of a node in the Random Waypoint Mobility Model is similar to the one in the Random Walk Mobility Model [27] when the pause time is zero. For more details about mobility models for Ad hoc networks see [30].

The nodes, initially placed uniformly and independently in the area, start moving according to the Random Waypoint Mobility Model. At the end of the simulation experiment, the different indices we are interested in are extracted from the output file.

- Mean number of neighbors:

  For a given node, we compute the number of its neighbors *nemb*. Repeating the same experiment Ω times, we obtain for the mean number

  $$\frac{\sum nemb}{\Omega}.$$

  Figure 6 shows the analytic and simulation results. For the initial value of $\lambda = 0.0015$, the two plots coincide. But when λ is increased (starting from $\lambda = 0.005$), we notice a small difference. This is because, by increasing the frequency, the number of messages using the available bandwidth increases, and thus the time that a Hello message takes to reach the destination node becomes longer. Some messages are lost because their sojourn time exceeds the TTL (Time to Live) [2].

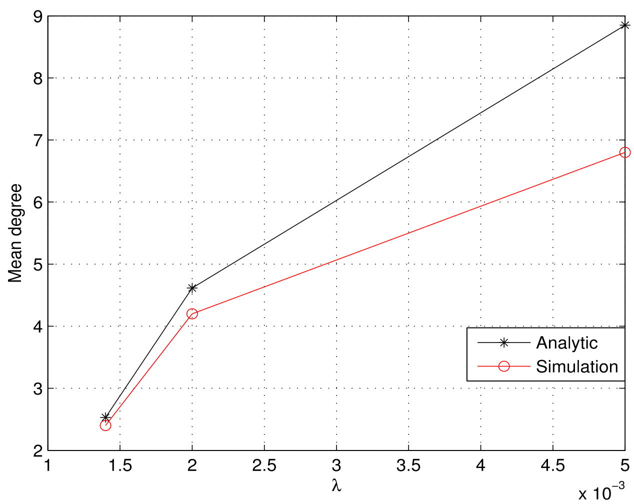


Fig. 6 Mean degree node

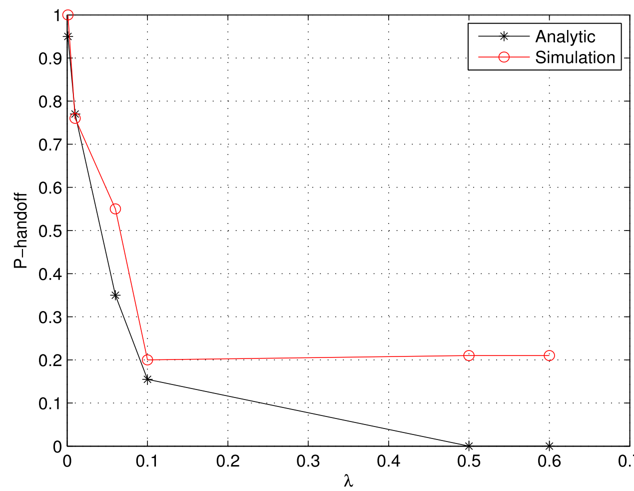


Fig. 8 Handoff probability

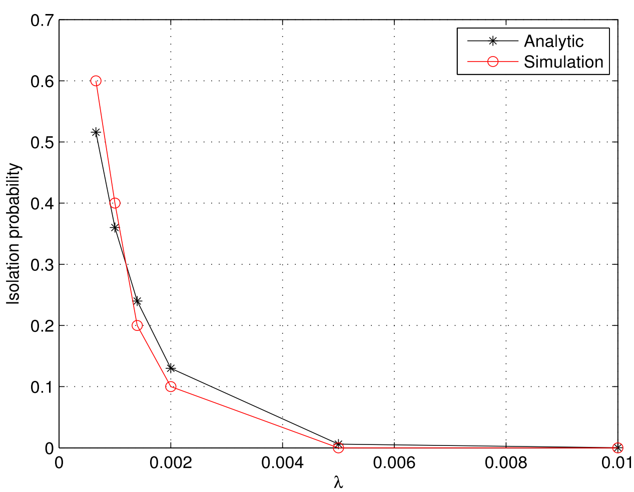


Fig. 7 Isolation probability

- Probability that a node is isolated:

  At the end of each simulation, we obtain the number of single nodes (the single node is an isolated one) *nembs*. The probability that a node is isolated is $nembs/(l+m)$. Repeating this experiment Ω times allows us to estimate correctly this probability (the probability that a node is isolated) to be

$$\frac{\sum(nembs/(l+m))}{\Omega}.$$

  Figure 7 illustrate the simulation results and those obtained by using formula (11).

  We see that there is a good agreement between the simulation results and the theoretical ones. When λ is small ($\lambda < 0.002$), the probability that a node is isolated is large, but as soon as λ becomes large ($\lambda > 0.008$), then this probability becomes almost zero. This can be explained by the following fact: since the frequency of receptions of the Hello messages is large, the knowledge of the neighborhood is good and the nodes can communicate.

- Handoff probability:

  A node which is not in the transmission range of its cluster-head tries to find another cluster-head and connects to it; this is a handover. The probability of this event is called the Handoff probability. Experimentally, the handover probability is taken, for a given node, as the ratio of the number of its handoff to the total number of hello messages. Figure 8 shows the simulation results and those obtained using formula (13).

  Again, we get a good agreement between the two plots. For the simulation results, as soon as $\lambda > 0.1$, the handover probability become constant. The reason is that the transmission frequency of hello messages do not affect the mobility of nodes, but the discovery of the neighborhood. The probability of a handoff is calculated as the ratio of the number of messages indicating a handoff on the total number of messages. then for ($\lambda < 0.1$), we have a loss of information (some handoff are not reported), but when ($\lambda > 0.1$) there are a better knowledge of the network and the probability of handoff is constant.

- Connectivity:

  The probability that a network is connected is computed as described in [5]. Taking $R = 250$ m and different values of λ, we place $l+m$ nodes in a large area. At the end of the simulation we check if the topologies are connected. The same experiment is repeated Ω times to compute the percentage of connected topologies

$$\frac{\text{number of connected topologies}}{\Omega},$$

  which gives a good estimate of $P(con)$.

  Figure 9 shows the simulation and analytic results for $P(con)$.

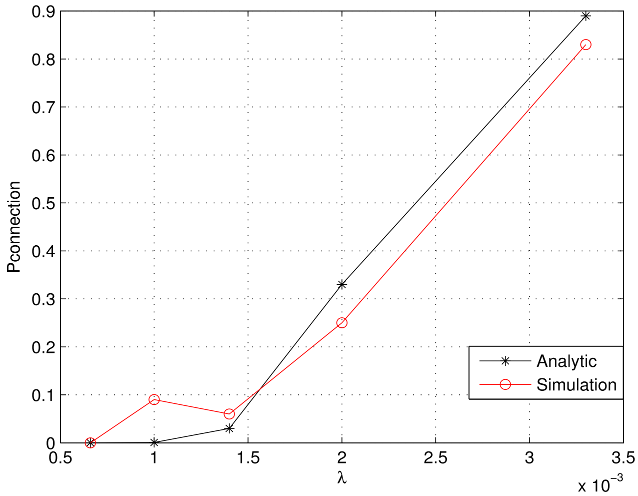


Fig. 9 Connectivity probability

Figure 9 tells us that using formula (9) was very appropriate. We can also see that $\lambda = 0.0035$ gives a probability, that the network is connected, larger than 0.8.

9 Conclusion

This work was motivated by the important impact that the transmission rate of Hello messages of an Ad hoc network had on its performances. We presented a modelling for an ordinary node positions and a cluster-head positions by two Markov Process $\{X_n\}$ and $\{Y_n\}$. We obtained results for the mean degree of a node, the isolation probability, the probability of a dominant set update, the probability of a handoff and the probability of connectivity $P(con)$ as a functions of the transmission frequency λ.

This study is part of an effort towards developing analytical results for the prediction of network connectivity using the information contained in Hello messages. Several future works could be drawn from our results. For instance, one could develop an adaptive algorithm for the management of the transmission frequency of Hello messages. One could also consider using a more general law to process the arrival of Hello messages, or consider the case where the speed of the nodes is arbitrary, or where the mobility region is bounded. It would also be interesting to extend this study by taking into account the presence of data messages. It is also interesting to consider the presence of data messages in the study. This could be used to improve the performance of routing protocols, gateway discovery algorithms [1], ...

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