Volume 2, N° 2, Décembre 1992 Prix : 35DA

ISSN 1111-0902

PREMIERE REVUE INTERNATIONALE ALGERIENNE DES TECHNOLOGIES AVANCEES

TECHNOLOGIES AVANCEES

SOMMAIRE

Recherche – Développement

Synthèse

TECHNOLOGIES AVANCEES

TECHNOLOGIES AVANCEES est une revue semestrielle éditée par le Centre de Développement des Technologies Avancées et traitant des thèmes suivants :

- Architecture des systèmes
- Génie logiciel.
- Intelligence artificielle et systèmes expert
- Systèmes d'information
- Automatique, robotique et productique
- Image et parole
- Traitement du signal
- Microélectronique
- Communication
- Télédétection
- Technologie et application des lasers
- Physique et application des plasmas
- Fusion thermonucléaire
- Biotechnologie

Centre de Développement des Technologies Avancées

128, Chemin Mohamed Gacem
El-Madania B.P. 245 Alger - Algérie
Tél : 67.73.25
Télex : 65.304
Fax : 65.33.60

ESTIMATE OF STRONG STABILITY IN THE SYSTEM

G/M/∞ .

Djamil AISSANI
Laboratory of Stochastic Modelisation - University of bejaia , Algeria

Abstarct : *Let Π_n denote the stationary distribution of a number of demands in a G/M/m system, consisdered at the time of the arrival of the next demand. Let denot the stationary distribution of the imbedded Markov chain in a G/M/oo system.*

In this paper ,we determine an estimate of the closeness between the stationary distributions Π_n and Π_n . (the perturbated system has the same properties as the G/M/∞ system).

Keys Words : *Estimation ,strong stability, uniform ergodicity, G/M/m system, G/M/∞ system, perturbation .*

I. INTRODUCTION

This paper is to be considered a continuation of [Aissani (87)]. We determine the conditions and estimations of the strong stability of chains describing the queue size (demands in service or waiting) in a G/M/∞ system, then, we find exact estimations to approximate characteristics of a G/M/m system by analogous characteristics of the system limit. For this, the perturbated system (the perturbation refers to the structure of the system) of type G/M/m has the same properties as the G/M/oo system.

The proofs are based on results [Kartashov (81) ,Aissani (85)] of the strong stability of Markov chains. To apply these results, we introduce in the second paragraph,a spécial class of norm opérators. In the third paragraph,we expose the results of this article .

Note that in practice parameters of queueing systems are, in general, known only approximately. The stability inequalities obtained can be used to estimate the error of defintion of the characteristics we are looking for, and this, after small perturbation of the system parameters.

2 . PRELIMINARIES AND NOTATIONS

Consider a G/M/m queueing system (FIFO, ∞) with H distribution of the period between the time of arrival of thedemands, and with exponential service time distributions with mean $\gamma^{1.}$

The transition chains are defined with the arrival of new demands and let X_n denote the number of demands (in service or waiting) being in the system at the time of the arrivel of the n-th demand. The sequence X_n constitutes a Markov chain with transition kernel $P_m = \|. P_{ij}(m)\|_{i,j=0}^{\infty}$. At the same time, we consider a G/M/∞ queueing system with infinite number of servers, having the same distribution as the G/M/m system considered previously. To estimate the difference between the stationary distributions of the chains X_n in the G/M/m and G/M/∞ systems, we introduce in the space $m = \{(\mu_n, n\geq 0)\}$ of finite measures on N, a family of norms,

$$\|\mu\|_v = \sum_{n\geq 0} v^n|\mu_n| \quad \text{for any} \quad V > 1$$

Where $\mu = (\mu_n , n\geq 0)$ and $|\mu|$ is the variation of the measure μ .

This norm induces a corresponding norm in the space of transition kernels on (N , B (N))

$$\|Q\|_v = \sup_{i\geq 0} V^{-i} \sum_{j\geq 0} V\,|Q_{ij}|$$

All the notions and notations not defined here, can be found in [9]. In particular, the defintions of uniform ergodicity and strong stability in the sense this research goes, are given in the first paragraph,where as the expressions of transition kernels P_m and $P_{\infty} = \|P_{ij}(\infty)\|_{i,j=0}^{\infty}$ are given in paragraph two, or for instance in [1] , [Karlin(68)] . Recall that we associate with each transition kernel P_{ij} the linear mapping $P_{ij}: m \longrightarrow m$ acting on $\mu \in m$ as follow :

$$(\mu P_s)_k = \sum_{j\geq 0} \mu_j . P_{jk}(s)$$

Secondly, we denote by μ (Z) and ν (Z) respectively the generating functions of the measures μ and ν . Furthermore, every integral with unspecified domain of integration is taken all over R+ .