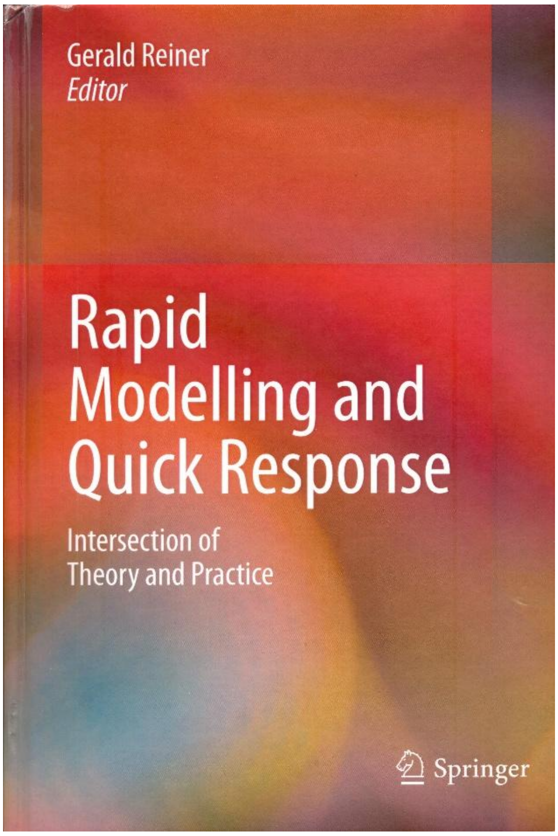
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Contents

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Perturbation Analysis of M/M/1 Queue

Karim Abbas and Djamil Aïssani

Abstract This paper treats the problem of evaluating the sensitivity of performance measures to changes in system parameters for a specific class of stochastic models. Motivated by the problem of the coexistence on transmission links of telecommunication networks of elastic and unresponsive traffic, we study in this paper the impact on the stationary characteristics of an M/M/1 queue of a small perturbation in the server rate. For this model we obtain a new perturbation bound by using the Strong Stability Approach. Our analysis is based on bounding the distance of stationary distributions in a suitable functional space.

Series Expansions in Queues with Server Vacation

Fazia Rahmoune and Djamil Aïssani

Abstract This paper provides series expansions of the stationary distribution of finite Markov chains. The work presented is a part of research project on numerical algorithms based on series expansions of Markov chains with finite state-space S. We are interested in the performance of a stochastic system when some of its parameters or characteristics are changed. This leads to an efficient numerical algorithm for computing the stationary distribution. Numerical examples are given to illustrate the performance of the algorithm, while numerical bounds are provided for quantities from some models like manufacturing systems to optimize the requirement policy or reliability models to optimize the preventive maintenance policy after modelling by vacation queuing systems.

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D. Aïssani, M. Cherfaoui , S. Adjabi, S. Hocine and N. Zareb

Abstract The BMT (Bejaia Mediterranean Terminal) Company of Bejaia's harbor became aware that the performances of the terminal with containers is measured by the time of stopover, the speed of the operations, the quality of the service and the cost of container's transit. For this end, the company has devoted several studies to analyze the performance of its terminal: elaboration of a global model for the "loaded / unloaded" process, modeling of the system by another approach which consists of the decomposition of the system into four independent subsystems (namely: the "loading" process, the "unloading" process, the "full-stock" process and the "empty-stock" process - (see Aïssani et al, 2009; Aïssani et al, 2009). The models used in this last study describe in detail the comportment of the real systems and the obtained results given by the simulators corresponding to each model are approximately the same as the real values. It is the reason for which the company wants to exploit these models in order to determine an optimal management of its equipments.

Indeed, this work consists, more specifically, in determining the optimal number of trucks to be used in each process that minimizes the waiting time of trucks and GQ (Gantry of Quay). This is a multi-objectives optimization problem, exactly a stochastic bi-objectives optimization problem. For that, we have modeled the problem by an open network which is the most suitable for this situation. After the identification of the process parameters, we conclude that the model is an open network of unspecified queues $(G^{[X]}/G/1,\ M/G/1,\ G/G/N/0, ...)$. In the literature, there is no exact method for analyzing this kind of networks. For this, we have established a simulation model that can imitate the functioning of each system.

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The simulations allowed us to evaluate the performances of the park with containers according to the number of the trucks used, on the basis of current conditions and in the case of variation of the flow of arrivals of ships and the service rate of the trucks. This allowed us to determine the optimal number of trucks to be used in the loading process and unloading process. We have also determined the performance of the stock, on basis of current conditions.

1 Introduction

The performance of a Terminal with containers is measured by the time of stopover, the speed of the operations, the quality of the service and the cost of container's transit. For that, in order to ensure the best functioning of the Terminal with containers at the level of the BMT Company (Bejaia Mediterranean Terminal - Bejaia's Harbor), studies of performance's evaluation were initiated. The first study was carried out in 2007 (Sait et al, 2007). It had for objective the global modeling of the unloading/loading process and had shown that if the number of ships [having a mean size of 170 ETU (Equivalent Twenty Units)], which was of 0.83 ships/day, increases to 1.4 ships/day, the full park will undergo a saturation of 94%. The second study was carried out in 2009 (Aïssani et al, 2009; Aïssani et al, 2009). It was intended to suggest an alternative approach for modeling the system. The authors proposed an approach which consists of decomposing the system into four independent subsystems, namely: the loading process, the unloading process, the full stock process and the empty stock process. The study showed that the park with containers has a possibility to handle 116226 ETU for an entry rate of 0.6104 ships / day for the loading process and 0.7761 ships/day for the unloading process.

The study showed also that for a 30% increase in the number of ships arriving at the port of Bejaia, we note a small increase in the average number of ships in roads and in the quays. On the other hand, there is a clear increasing in the total number of treated containers which passes from 116226 ETU to 148996 ETU. We note also an increase in the average number of containers in the full park which passes from 3372 to 4874 ETU. As for the number of treated ships, it passes from 240 to 305 ships at the loading and from 296 to 382 ships at the unloading.

In the present work, we propose to supplement this last study where we try to minimize the number of trucks to use in the treatment of the ships. The interest of this analysis comes from the fact that the permanent increase of traffic constrains the BMT Company to exploit other quays. To this end, in order to optimize the existing equipments, the problem will be modeled by an open network, which belongs to the multi-objectives optimization problems. Because of the complexity and the unavailability of analytical methods for analyzing this type of models, we apply the simulation approach.

2 Park with containers and motion of the containers

In this section, we are going to give a brief description of the Terminal of BMT Company, where we will present different operations and movements of a container and its capacities and equipments.

2.1 Motions of the containers

Any container (ship) arriving at the Terminal of BMT Company passes by the following steps:

- **The step of anchorage:** Any ship arriving at the Bejaia harbor's is put on standby in the anchorage (roads) for a duration of time which varies from one ship to another, because of the occupation of the quay stations or unavailability of pilot or tug boats.
- **The step of service:**
  - **Service of accosting:** The accosting of the ships is ensured by the operational section of the Harbor Company of Bejaia, such as the section of piloting and towing.
  - **Vessel handling:** the treatment of a ship is done mainly in three sub-steps:
    1. **Service before operations:** It is the preparatory step of the ship for the handling (Loading/Unloading).
    2. **Step of Unloading/Loading:** It consists of the unloading/loading of the containers. This is carried out with the two gantries of quay which have carriages being able to raise the containers from the container ships, to put them on trucks and to raise the container from the trucks and put them on board the container ship, if it's the loading process.
    3. **Service after operations:** It is the preparatory step of the ship for accosting towards outside.
- **Deliveries:** The delivery concerns the full containers or discharged goods. The means used to perform this operation are: RTG (Rubber Tyre Gantry), trucks, stacker and forklifts if it's necessary.
- **Restitution of the containers:** At the restitution of the containers (empty containers), two zones are intended for the storage, one for empty containers of 20 units and the other for empty containers of 40 units.

2.2 The BMT Park with containers: capacity and equipments

The Terminal of the BMT Company is provided with four quays of 500 m (currently only two are in the exploitation), a draught of 12 *m* starting from the channel, and a storage capacity of 10300 ETU, the Terminal with containers of Bejaia offers specialized installations for the refrigerating containers and the dangerous products. Moreover, this Terminal is the only Terminal with containers in Algeria, sufficiently equipped and has specialized equipments (Gantry of Quay, RTG . . .), handling and lifting, which can reduce the times of stopover, making it possible to fulfill waiting and the requirements of the operators (See Table 1).

Quay /Anchorage	Length:	500 m
	Depth:	12 m
	Basin surface:	60 h
	Quay:	4
	Utilisation rate of the quay	70%
Full Park	Capacity:	8300 ETU
	Area:	68500 m^2
Empty Park	Capacity:	900 ETU
	Area:	15200 m^2
Refrigerating Park	Capacity:	500 Catches
	Area:	2800 m^2
Zone for Discharge / Potting	Capacity:	600 ETU
	Area:	3500 m^2

Gantry of Quay	Numbers:	2
	Tonnage:	40 Tons
	Type:	Post Panamax
RTG (Rubber Tyre Gantry)	Numbers:	5
	Tonnage:	36 Tons
	Stacking:	6+1 on the ground and 4+1 in Height
Stakers	Numbers:	4
	Tonnage:	36 Tons
Spreaders	Numbers:	4
	Tonnage:	10 Tons
Lifting trucks	Numbers:	02 of 03 Tons, 02 of 05 Tons 02 of 10 Tons and 02 of 28 Tons
Truck-Tug	Numbers:	8 of 60 Tons and 4 of 32 Tons

Table 1 Characteristics and equipments of the Terminal of BMT Company.

3 Mathematical Models

After analyzing the main movements of a container at the level of BMT's Terminal, we chose to model the problem by network, which is most suitable for this type of situation. To this effect, we obtained four models, namely: the empty stock, the full stock, the loading and the unloading processes which are given respectively by Figures 1 and 2.

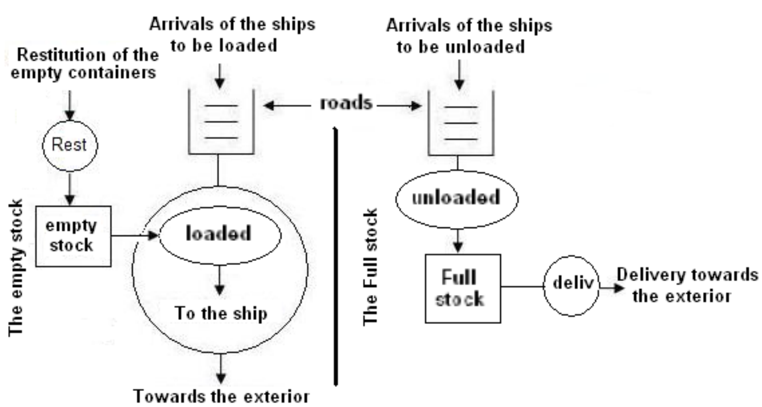

Fig. 1 Diagram of the models of the storage processes.

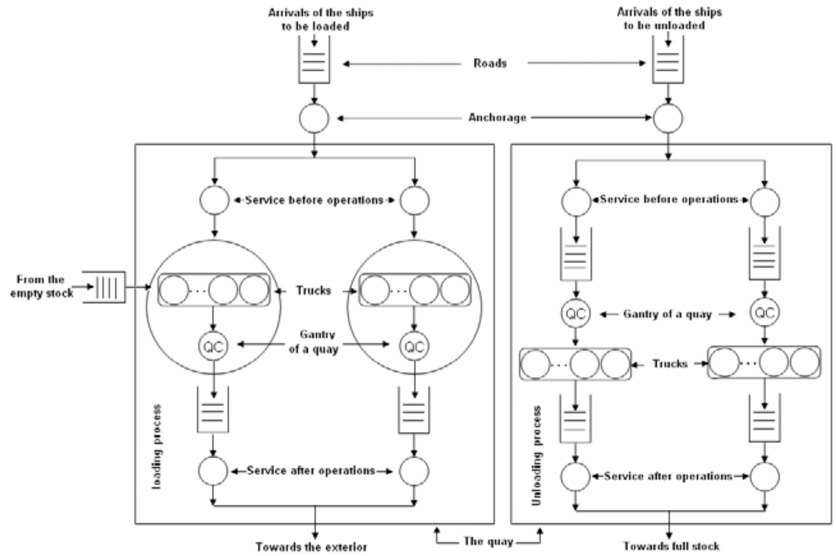

Fig. 2 Diagram of the models of the ship's treatments (unloaded/loaded process).

4 Calculation of the Forecasts

In February 2009, a calculation of forecast had been carried out. The designed series is the number of containers treated (loaded/unloaded) in ETU. The data used were collected monthly and were held forth over a period of two years (from January 2006 to February 2009). The method used for calculation of the forecasts is the exponential smoothing method (David and Michaud, 1983). Figure 3 and Table 2 represent the original series of the number of containers in ETU, as well as the forecasts (from March to December 2009). Thus, it is noted that the objective that BMT Company had fixed at the beginning of the year was likely to be achieved.

5 Performance Evaluation of the BMT Terminal

First, we will conduct a statistical analysis to identify the network corresponding to our system.

Months	Historic				Forecast
	2006	2007	2008	2009	
January	4938	6102	9695	10066	_
February	6006	10083	9928	11448	_
March	6445	8565	9882	_	11579.74
April	5604	9535	8791	_	11941.29
May	6519	8938	10155	_	12314.13
June	5909	8337	8799	_	12698.61
July	6041	7582	9338	_	13095.09
August	7552	7245	9304	_	13503.96
September	5915	8135	9171	_	13925.59
October	5938	7982	8779	_	14360.38
November	7858	7579	10984	_	14808.75
December	7636	9971	11596	_	15271.12
Total					133498.7

Table 2 Original series and forecasts of the number of containers to be treated (ETU) in the year 2009.

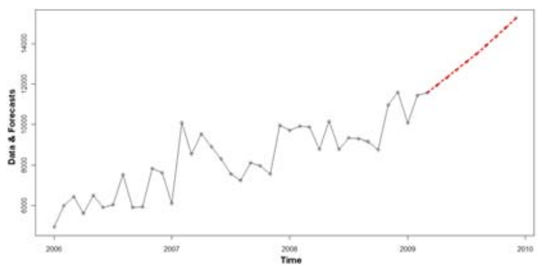

Fig. 3 Graph of original series and forecasts of the number of containers to be treated (ETU) in the year 2009.

5.1 Statistical Analysis and Identification of Models

The results of the preliminary statistical analysis (estimate and test adjustment) on the collected data for the identification of process parameters are summarized in Table 3.

According to this preliminary analysis, we conclude that the performance evaluation of the Terminal of Bejaia is really a complex problem. Indeed, the system is modeled by an opened network of unspecified queues, because it consists of queues of type $(G^{[X]}/G/1,\ M/G/1,\ G/G/N/0$, with blocking,...). Therefore, we cannot use analytical methods (as for the Jackson networks or BCMP) to obtain the characteristics of the system. This is why we will call upon the simulation approach to solve the problem.

Process	Variable	Distribution	Parameters of the distribution
Loading	Inter-arrivals of the ships to be loaded (minutes)	Exponential	$\lambda = 2710$
	service duration of the anchorage (minutes)	Normal	$\mu = 57.595$ et $\sigma^2 = 18.174$
	service duration of before operations (minutes)	Normal	$\mu = 99.175$ et $\sigma^2 = 38.678$
	Size of groups to be loaded	Geometric	$p = 0.0059$
	service duration of the gantries of quay (minutes)	Normal	$\mu = 2.944$ et $\sigma^2 = 1.097$
	Service duration of the trucks (minutes)	Normal	$\mu = 8.823$ et $\sigma^2 = 5.359$
	service duration of the after operations (minutes)	Normal	$\mu = 99.175$ et $\sigma^2 = 38.678$
Unloading	Inter-arrivals of the ships to be unloaded (minutes)	Exponential	$\lambda = 2710$
	service duration of the anchorage (minutes)	Normal	$\mu = 57.595$ et $\sigma^2 = 18.174$
	service duration of before operations (minutes)	Normal	$\mu = 99.175$ et $\sigma^2 = 38.678$
	Size of groups to be loaded	Geometric	$p = 0.007$
	Service duration of the gantries of quay (minutes)	Normal	$\mu = 2.947$ et $\sigma^2 = 1.072$
	Service duration of the trucks (minutes)	Normal	$\mu = 9.228$ et $\sigma^2 = 4.994$
	service duration of the after operations (minutes)	Normal	$\mu = 99.175$ et $\sigma^2 = 38.678$
Storage	Size of groups of delivered containers/day	Uniform	Mean=145
	Storage Size of groups of restored containers/day	Uniform	Mean=140

Table 3 Results of the statistical analysis on the collected data.

5.2 Determination of the optimal number of trucks by simulation

In this section, the aim is to determine by simulation approach the optimal number of trucks to use during the loading process and unloading process. For that, we propose two approaches.

5.2.1 First approach

We designed a simulator for each model under the Matlab environment. After the validation tests of each simulator, their executions provided the results summarized in Table 4.

Where:

- The 3rd column represents the mean number of ships in roads to be loaded (respectively to be unloaded) during one year.
- The 4th column represents the mean number of ships loaded (respectively unloaded) during one year.
- The 5th column represents the mean number of containers loaded (respectively unloaded) during one year.
- The 6th column represents the mean number of blocking of the server "GQ" according to the number of trucks used during the loading (respectively the unloading) on one year.
- The 7th column represents the mean time of blocking of the server "GQ" during the loading (respectively the unloading) on one year.
- The 8th column represents the mean number of blocking of the server "trucks" in the loading process (respectively the unloading) on one year.

Process	N-trucks	N-ship	D-ship	N-Cts (ETU)	N-GQ	W-GQ	N-trucks	W- trucks	Proportions
Loading	1	1.20	191.09	49020	27495	3349.5	2927	72.6	0.0909
	2	0.87	193.88	49397	18804	1118.1	11878	342.1	0.3642
	3	0.88	193.74	49681	11775	449.2	18797	588.3	0.5757
	4	0.79	196.08	50204	7150.8	196.7	22876	761.4	0.7094
	5	0.78	195.63	50298	4749.0	103.8	25723	899.6	0.7818
	6	0.85	196.59	50753	3442.5	66.3	27543	1001.7	0.8190
	7	0.83	196.00	50090	2577.0	47.5	27453	1030.7	0.8365
	8	0.89	194.59	49577	2079.3	38.1	27699	1068.1	0.8463
	9	0.84	195.64	50233	1725.9	32.2	27428	1080.7	0.8496
	10	0.93	194.34	49717	1548.9	29.3	28240	1133.8	0.8517
	11	0.84	193.90	49260	1316.9	25.6	26870	1095.6	0.8509
	12	0.84	192.93	49187	1268.9	25	28594	1181.4	0.8512
Unloading	1	0.90	192.13	41689	24112	2955.5	2099	50.40	0.0770
	2	0.90	191.57	41523	15930	947.7	9289	259.90	0.3418
	3	0.73	193.00	42321	9035	342.8	13713	406.95	0.4952
	4	0.80	197.20	42093	3731.5	102.4	15794	435.37	0.5734
	5	0.87	196.70	41971	1025.2	21.6	16544	516.73	0.6023
	6	0.73	191.33	41087	166.83	2.9	16386	548.40	0.6095
	7	0.90	199.60	42798	19.80	0.3	17103	537.75	0.6103
	8	0.77	195.13	42548	1.50	0	17002	545.60	0.6106
	9	0.77	192.97	40966	0.10	0	16378	549.78	0.6109
	10	0.83	194.90	43800	0	0	17525	551.85	0.6115
	11	0.70	194.40	43401	0	0	17359	546.22	0.6111
	12	0.73	192.97	41572	0	0	16607	539.27	0.6104

Table 4 Some performances of the processes obtained by simulation approach.

- The 9th column represents the total mean time of blocking of the server ”trucks” in the loading (respectively the unloading) on one year.
- The 10th column represents the probabilities of the blocking of the servers ”trucks” in the loading process (respectively the unloading process); for example: the value 0.5757 of the third row represents the blocking probability in the case of three servers ”trucks” in the loading process, which is the sum of the probabilities of blocking of one server, two servers and three servers.
  These probabilities is distributed as following: $P(X=0)=0.4276$,
  $P(X=1)=0.4117$, $P(X=2)=0.1492$, $P(X=3)=0.0148$,
  where X: ’number of servers ”trucks” blocked and
  $P(X=1)+P(X=2)+P(X=3)=0.5757$. This distribution is illustrated by the figure (left).

Interpretation and discussion of the results

- **Loading process**

- From the obtained results, we note that the variation of the mean number of the loaded containers in ETU during one year is independent of the number of trucks used . Indeed, the mean number of the loaded containers varies only between

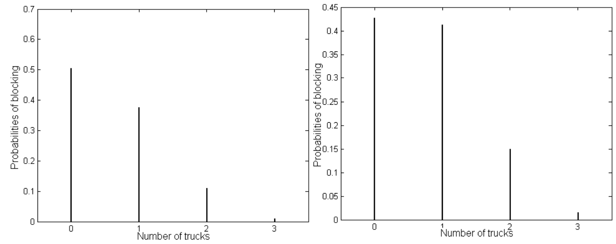

Fig. 4 Probabilities of blocking of the servers "trucks": case of three trucks (loading process on the left and unloading process on the right).

49019.5401 ETU and 50753.3083 ETU which is practically the same. This independence can be explained by the fact that the inter-arrivals of ships to be loaded are very large compared to the time spent by ship at the quay. Similarly, for the loading process, the mean number of ships on the roads is also independent of the number of trucks used, except in the case of one truck, where the mean number of ships in roads is a bit high (1.2000 ships). But, according to the dashed curve, we note that the mean waiting time of the trucks (blocking of servers "trucks") is proportional to the number of the used servers "trucks". So, to minimize the blocking duration of the servers "trucks", we must use the minimum possible of servers "trucks".

- This problem can be formulated (written) mathematically as follows:

$$
\begin{array}{lcl}
\begin{array}{l} \min \longleftarrow T_1, \\ \min \longleftarrow T_2, \end{array} & & \min \longleftarrow (T_1, T_2) \\
S.C. \left\{ \begin{array}{l} \text{Capacities of the company,} \\ \text{Available equipments,} \\ \text{Processing time.} \end{array} \right. & \textit{or} & S.C. \left\{ \begin{array}{l} \text{Capacities of the company,} \\ \text{Available equipments,} \\ \text{Processing time.} \end{array} \right.
\end{array} \quad (1)
$$

Where T_1 and T_2 represent respectively the mean waiting time of the server "GQ" and the mean waiting time of the servers "trucks". Here, we note that we are facing a stochastic multi-objectives optimization problem, specifically a stochastic bi-objectives problem. So, to determine the optimal number of trucks to use in the loading process, it is necessary to find a compromise between the blocking time of the server "GQ" and the blocking time of the servers "trucks". It is thus necessary to find the number of servers "trucks" which minimizes the blocking time of the server "GQ" and minimizes the blocking time of the servers "trucks" at the same time.

So, we transform the problem (1) to the following form Weighted Sum Scalarization (see Ehrgott, 2005; Bot et al, 2009):

$$\min \longleftarrow \alpha T_1 + (1-\alpha) T_2,$$

$$S.C. \begin{cases} \text{Capacities of the company,} \\ \text{Available equipments,} \\ \text{Processing time.} \end{cases} \quad (2)$$

Where: α is weight reflecting the preference of the waiting time of the server "GQ" and $1-\alpha$ the weight reflecting the preference the waiting time of the servers "trucks". In this work, we assume that there is not a preference between the waiting of the servers "trucks" and the waiting of server "GQ" i.e $\alpha = 0.5$. In this case, we determine the minimum of the sum of the blocking times of the server "GQ" and servers "trucks" which is represented by the solid curve on the Figure 5 (left). According to this curve, we note that the optimal number of trucks to be used (which minimize the sum of the blocking times for loading process) is four (04) trucks.

- **Unloading process:** With the same manner and same reasoning as in the loading process, we can determine the optimal number of trucks that will be used in the unloading process. In this case, the result is also four (04) trucks.

5.2.2 Second approach

In this part, we propose another approach (reasoning) to determine the optimal number of trucks to be used. This method consists of determining the number of trucks to use in order to minimize the mean time of loading or unloading of a ship (beginning operations - end operations). The obtained results for different number of servers "trucks" used are summarized in the table 5.

Number of trucks	1	2	3	4	5	6
Loading service	26.9163	13.9531	10.3773	8.2871	8.2671	8.4758
Unloading service	23.8718	13.4766	10.2773	9.1891	9.0743	9.3003
Number of trucks	7	8	9	10	11	12
Loading service	8.3842	8.4986	8.4049	8.4796	8.4955	8.3235
Unloading service	9.1870	8.9528	8.6114	8.9666	9.0482	8.6716

Table 5 The Variation of the mean time (hours) of the loading/unloading service, according to the number of servers "trucks".

Interpretation and discussion of results

Loading process: Figure 5 (right) and the second row of the Table 5 show that the mean time of loading service decreases with the number of servers "trucks" from (01) to four (04) servers "trucks", and from four (04) trucks, the mean time of loading service remains almost constant, which means that beyond four (04) servers

”trucks”, the mean time of loading service depends only on the ability of the server ”GQ”. To this end, we conclude that we no interest to use more than four (04) servers ”trucks” in the loading process.

So the optimal number of trucks towing in this case is four (04) trucks.

Unloading process : for the same arguments as the loading process, the optimal number for the unloading process is four (04) trucks.

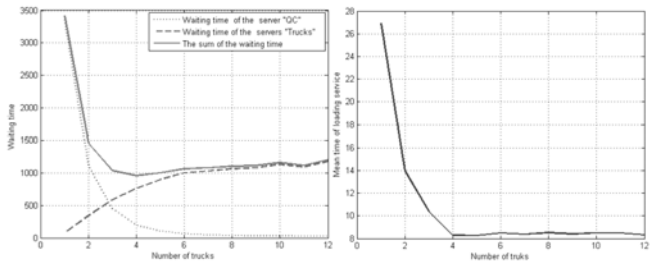

Fig. 5 The mean waiting time of the servers ”trucks” and ”GQ” on one year (left) and he variation of the mean time of loading service (right) according to the number of servers ”trucks”.

6 Performance study of storage process

After the validation tests of empty stock and full stock simulators, their executions provided the results summarized in Table 6. Where the 2nd, 3rd and 4th column rep-

Parameters	Number of trucks	ETU	saturation (%)
Full stock	4	4570.9995	55.0722
	5	4440.8319	53.5044
Empty stock	4	1157.5036	128.6115
	5	1192.9385	132.5487

Table 6 Storage performances.

resents respectively the number of servers ”trucks” used, the total mean number of containers (ETU) in the full stock and empty stock on one year and their saturation rate expressed as a percentage.

Interpretation and discussion of the results

The simulation results show that:

- With the current parameters, the average number of containers in the full park over a period of one year is 4570.9995 ETU in the case of four (04) ”trucks”, and 3610.8734 ETU in the case of five (05) ”trucks” and the mean number of containers in the empty park over a period of one year is 1157.5036 ETU in the case of four (04) ”trucks” and 1192.9385 ETU in the case of five (05) servers ”trucks”.

7 Conclusion

The objective of this work is to determine an optimal management of the equipments of the Terminal with containers of the BMT Company, more specifically the optimal number of trucks to use in the loading process and unloading process. For this, we developed a mathematical model for each process (the ”loading”, the ”unloading”, the ”full stock” and the ”empty stock” process). Indeed, in order to analyze the different processes and determine the optimal number of trucks to use, each system (process) is modeled by an open network. We have also established a simulation model of each system, where the goal of each simulator is to reproduce the functioning of the park with containers. The study shows that:

- For the loading process: For an arrival rate of 0.5317 $ships/day$, a mean service trucks of 8.8234 $minutes$ and a mean service GQ of 2.9440 $minutes$, the optimal number of trucks is four (04) trucks. This mean that the BMT Company can recover a truck from each GQ, i.e in total two (02) trucks.
- For the unloading process: For an arrival rate of 0.5317 $ships/day$, a mean service trucks of 9.2281 $minutes$ and a mean service GQ of 2.9473 $minutes$, the optimal number of trucks is four (04). This mean that the BMT Company can recover a truck from each GQ, i.e in total two (02) trucks.
- Regarding the stock: The study shows that for the current settings at the end of the year 2009 it will undergo a saturation of 55% for the full stock and 130% for the empty stock, hence the need for expanding the capacity of empty stock.

It would be interesting to achieve this work, by discussing the following items:

- An analytical resolution of the problem.
- Determination of an optimal management of the others equipments of the BMT Company.
- Take account the variation of the parameters of the system.

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Perturbation Analysis of M/M/1 Queue

Karim Abbas and Djamil Aïssani

Abstract This paper treats the problem of evaluating the sensitivity of performance measures to changes in system parameters for a specific class of stochastic models. Motivated by the problem of the coexistence on transmission links of telecommunication networks of elastic and unresponsive traffic, we study in this paper the impact on the stationary characteristics of an M/M/1 queue of a small perturbation in the server rate. For this model we obtain a new perturbation bound by using the Strong Stability Approach. Our analysis is based on bounding the distance of stationary distributions in a suitable functional space.

1 Introduction

A manufacturing process or a telecommunication network is a dynamical system which can, in principle, be described by using a rapid modelling technique such as queueing theory. In particular, the usefulness of the M/M/1 queueing model is multiplied many times if the model approximates the behaviour of queues that deviate slightly from the assumptions in the model. An analysis of the sensitivity of performance measures to changes in model parameters is an issue of practical importance. The study of this queueing model is motivated by the following engineering problem: Consider a transmission link of a telecommunication network carrying elastic traffic, able to adapt to the congestion level of the network, and a small proportion of traffic, which is unresponsive to congestion. The problem addressed in this paper

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is to derive quantitative results for estimating the influence of unresponsive traffic on elastic traffic.

Suppose, for example, we wish to model the behaviour of a single server queue with an infinite waiting room and a first-in first-out discipline. Let P denote the transition kernel of the imbedded jump chain of the M/M/1 queue with arrival rate λ and service rate μ. This Markov chain, defined on some (denumerable) state-space S, has unique stationary distribution π. What would be the effect on the stationary performance of the queue if we increased the service rate of the server by ε? Let $\widetilde{P}$ denote the Markov transition kernel of the Markov chain modeling the alternative system, in our example the M/M/1 queue with service rate $\mu+\varepsilon$, and assume that $\widetilde{P}$ has unique stationary distribution $\widetilde{\pi}$. The question about the effect of switching from P to $\widetilde{P}$ on the stationary behavior is expressed by $\pi-\widetilde{\pi}$, the difference between the stationary distributions. Obviously, a bound on the effect of the perturbation is of great interest. More specifically, let $\|\cdot\|_{\mathrm{tv}}$ denote the total variation norm, then the above problem can be phrased as follows: Can $\|\pi-\widetilde{\pi}\|_{\mathrm{tv}}$ be approximated or bounded in terms of $\|P-\widetilde{P}\|_{\mathrm{tv}}$? This is known as *"perturbation analysis" or "stability problem" of Markov chains* in the literature. However, convergence w.r.t. to the total variational norm allows for only bounding the effect of switching from P to $\widetilde{P}$ for bounded performance measures only.

There exists numerous results on perturbation bounds of Markov chains. General results are summarized by Heidergott and Hordijk (2003). One group of results concerns the sensitivity of the stationary distribution of a finite, homogeneous Markov chain (see Heidergott et al, 2007), and the bounds are derived using methods of matrix analysis; see the review of Cho and Meyer (2001) and recent papers of Kirkland (2002) and Neumann and Xu (2004). Another group includes perturbation bounds for finite-time and invariant distributions of Markov chains with general state space; see Anisimov (1988), Rachev (1989), Aïssani and Kartashov (1983), Kartashov (1996) and Mitrophanov (2005). In these works, the bounds for general Markov chains are expressed in terms of ergodicity coefficients of the iterated transition kernel, which are difficult to compute for infinite state spaces. These results were obtained using operator-theoretic and probabilistic methods.

Recent work examines the robustness of queueing models in a general framework. The sensitivity of the queue to a perturbation is measured by various functions (metrics) of the probability distributions associated with the perturbed and nominal queueing processes. These analyses can be found in Kotzurek and Stoyan (1976), Whitt (1981), Zolotarev (1977), Fricker et al (2009) and Altman et al (2004). Related work on the robustness of statistical models can be found in Albin (1982). In Albin (1984), the author has examined the robustness of the $M/M/1$ queueing model to several specific perturbations in the arrival process. The author has used the Taylor series expansion to predict accurately the operating characteristics of queues with arrival processes that are slightly different from the Poisson process. More recently, the $M/M/1$ queueing model with perturbations in the service process, has been studied via a perturbation analysis of a Markov chain by Heidergott (2008). In this paper, we consider the same perturbation introduced in Heidergott (2008), and obtain a new results by applying an other approach. Therefore, we set out to explain our approach with the M/M/1 queue with service rate μ as P and the M/M/1 queue with service rate $\mu+\varepsilon$ as the $\widetilde{P}$ system, for ε sufficiently small. For these systems π and $\widetilde{\pi}$ are known and everything can be computed explicitly. This allows for evaluating the potential of our approach.

The paper is organized as follows. Section 2 presents the Strong Stability Approach. Section 3 is devoted to establish the bound on the perturbation. Numerical examples are presented in Section 4. Eventually, we will point out directions of further research.

2 Strong Stability Approach

The main tool for our analysis is the weighted supremum norm, also called υ-norm, denoted by $\|\cdot\|_\upsilon$, where υ is some vector with elements $\upsilon(s)>1$ for all $s\in S$, and for any $w\in\mathbf{R}^{\mathscr{S}}$

$$\|w\|_\upsilon=\sup_{i\in\mathscr{S}}\frac{|w(i)|}{\upsilon(i)}.$$

Let μ be a probability measure on S, then the υ-norm of μ is defined as

$$\|\mu\|_\upsilon=\sum_{s\in S}\upsilon(s)\mu(ds).$$

The υ-norm is extended to operators on S in the following way: let $A\in\mathbf{R}^{S\times S}$ then

$$\|A\|_\upsilon=\sup_{i,\|w\|_\upsilon\le 1}\frac{\sum_{j=1}^{S}|A(i,j)w(j)|}{\upsilon(i)}.$$

Note that υ-norm convergence to 0 implies elementwise convergence to 0.

Suppose that $\widetilde{\pi}$ and π have finite υ-norm, then

$$|\widetilde{\pi}f-\pi f|\le\|\widetilde{\pi}-\pi\|_\upsilon\,\|f\|_\upsilon\inf_{s\in S}\upsilon(s),$$

for all f with $\|f\|_\upsilon$ finite. For our analysis, we will have $S\subset\mathbf{N}$ and we will assume that $\upsilon(s)$ is of particular form $\upsilon_\beta(s)=\beta^s$, for $\beta>1$. Hence, the bound becomes

$$|\widetilde{\pi}f-\pi f|\le\|\widetilde{\pi}-\pi\|_{\upsilon_\beta}\,\|f\|_{\upsilon_\beta}, \tag{1}$$

for all f such that $|f(s)|\le c\beta^s$ for some finite number c.

Denote the stationary distribution of P by π (and the stationary projector of P by Π) and denote the stationary distribution of $\widetilde{P}$ by $\widetilde{\pi}$ (and the stationary operator of $\widetilde{P}$ by $\widetilde{\Pi}$). Let

$$D=\sum_{n\ge 0}(P^n-\Pi).$$

Elementary calculation shows

$$(I-P)D = I - \Pi.$$

Multiplying this equation by $\widetilde{\Pi}$ and using the fact that it holds that $\widetilde{\Pi}\Pi = \Pi$ yields

$$\widetilde{\Pi}(I-P)D = \widetilde{\Pi} - \Pi.$$

Using that $\widetilde{\Pi} = \widetilde{\Pi}\widetilde{P}$, we obtain

$$\widetilde{\Pi} = \Pi + \widetilde{\Pi}(\widetilde{P} - P)D. \tag{2}$$

Inserting the righthand side of the above expression repeatedly for $\widetilde{\Pi}$ yields

$$\widetilde{\Pi} = \Pi \sum_{n\geq 0}^{\infty} ((\widetilde{P} - P)D)^n. \tag{3}$$

Switching from matrix to vector notation we arrive at the following basic series expansion

$$\widetilde{\pi} = \pi \sum_{n\geq 0}^{\infty} ((\widetilde{P} - P)D)^n. \tag{4}$$

We say that the Markov chain X with transition kernel P verifying $\|P\|_\upsilon < \infty$ and invariant measure π is *strongly υ-stable*, if every stochastic transition kernel $\widetilde{P}$ in some neighborhood $\{\widetilde{P} : \|\widetilde{P} - P\|_\upsilon < \varepsilon\}$ admits a unique invariant measure $\widetilde{\pi}$ such that $\|\widetilde{\pi} - \pi\|_\upsilon$ tends to zero as $\|\widetilde{P} - P\|_\upsilon$ tends to zero. The key criterion of strong stability of a Markov chain X is the existence of a deficient version of P defined in the following.

Let X be a Markov chain with the transition kernel P and invariant measure π. We call a deficient Markov kernel T a *residual for P with respect to* $\|\cdot\|_\upsilon$ if there exists a measure σ and a nonnegative measurable function h on $\mathbf{N}$ satisfying the following conditions:

a. $\pi h > 0$, $\sigma \mathbf{1} = 1$, $\sigma h > 0$, and
b. the operator $T = P - h \circ \sigma$ is nonnegative,
c. the norm of the operator T is strictly less than one, i.e., $\|T\|_\upsilon < 1$,
d. $\|P\|_\upsilon < \infty$,

where $\circ$ denotes the convolution between a measure and a function and $\mathbf{1}$ is the vector having all the components equal to 1.

It has been shown in Aïssani and Kartashov (1983) that a Markov chain X with the transition kernel P is strongly stable with respect to υ if and only if a residual for P with respect to υ exists. Although the strong stability approach originates from stability theory of Markov chains, the techniques developed for the strong stability approach allow to establish numerical algorithms for bounding $\|\widetilde{\pi} - \pi\|_\upsilon$ (Abbas and Aïssani, 2010a,b; Bouallouche-Medjkoune and Aïssani, 2006; Rabta and Aïssani, 2005). To see this, revisit the series expansion in (4). The following equality has been established independently by Kartashov (1986) and Hordijk and Spieksma (1994):

$$D = (I - \Pi) \sum_{n \geq 0} T^n (I - \Pi). \tag{5}$$

It is worth noting that for the above relation to hold it is necessary that T is a residual with respect to P. Note that $(\widetilde{P} - P)(I - \Pi) = (\widetilde{P} - P)$ and multiplying (5) by $(\widetilde{P} - P)$ yields

$$(\widetilde{P} - P)D = (\widetilde{P} - P) \sum_{n \geq 0} T^n (I - \Pi).$$

Inserting the above into (3) yields

$$\widetilde{\pi} - \pi = \pi \sum_{n \geq 1}^{\infty} \left((\widetilde{P} - P) \sum_{m \geq 0} T^m (I - \Pi) \right)^n. \tag{6}$$

Based on this series expansion, a bound on $\|\widetilde{\pi} - \pi\|_\upsilon$ is established in the following theorem.

Theorem 0.1. *(Kartashov, 1986) Let P be strongly stable. If*

$$\|\widetilde{P} - P\|_\upsilon < \frac{1 - \|T\|_\upsilon}{\|I - \Pi\|_\upsilon}$$

then, we the following bound holds

$$\|\widetilde{\pi} - \pi\|_\upsilon \leq \|\pi\|_\upsilon \frac{\|I - \Pi\|_\upsilon \, \|\widetilde{P} - P\|_\upsilon}{1 - \|T\|_\upsilon - \|I - \Pi\|_\upsilon \, \|\widetilde{P} - P\|_\upsilon}.$$

Proof. Under the assumptions of the theorem, it holds that

$$\begin{aligned} \left\| (\widetilde{P} - P) \sum_{m \geq 0} T^m (I - \Pi) \right\|_\upsilon &\leq \|(\widetilde{P} - P)\|_\upsilon \left\| \sum_{n \geq 0} T^n \right\|_\upsilon \|I - \Pi\|_\upsilon \\ &\leq \|(\widetilde{P} - P)\|_\upsilon \frac{\|I - \Pi\|_\upsilon}{1 - \rho}, \end{aligned}$$

where we use the fact that strong stability implies that $\|T\|_\upsilon < 1$. Provided that

$$\|(\widetilde{P} - P)\|_\upsilon < \frac{1 - \|T\|_\upsilon}{\|I - \Pi\|_\upsilon},$$

the series in (6) converges in υ-norm sense and we obtain with the help of the above inequality that

$$\left\| \sum_{n \geq 0}^{\infty} \left((\widetilde{P} - P) \sum_{m \geq 0} T^m (I - \Pi) \right)^n \right\|_\upsilon \leq \frac{1 - \|T\|_\upsilon}{1 - \|T\|_\upsilon - \|(\widetilde{P} - P)\|_\upsilon \|I - \Pi\|_\upsilon},$$

which proves the claim.

Note that the term $\|I-\Pi\|_{\upsilon}$ in the bound provided in Theorem 0.1 can be bounded by

$$\|I-\Pi\|_{\upsilon} \leq 1+\|\mathbf{1}\|_{\upsilon}\|\pi\|_{\upsilon}.$$

3 Analysis of the Model

We first consider the M/M/1 queue with arrival rate λ and service rate μ. Let $\rho = \lambda/\mu$. Markov kernel P is then given by

$$P_{ij} = \begin{cases} \frac{\lambda}{\lambda+\mu} & j = i+1, \\ \frac{\mu}{\lambda+\mu} & j = i-1, \ i > 0, \\ 0 & \text{otherwise}, \end{cases}$$

and $P_{00} = 1-\lambda/(\lambda+\mu)$. The kernel $\widetilde{P}$ of the M/M/1 queue with the service rate $\mu+\varepsilon$, such that $\mu+\varepsilon > \lambda$, is given by

$$\widetilde{P}_{ij} = \begin{cases} \frac{\lambda}{\lambda+\mu+\varepsilon} & j = i+1, \\ \frac{\mu+\varepsilon}{\lambda+\mu+\varepsilon} & j = i-1, \ i > 0, \\ 0 & \text{otherwise}, \end{cases}$$

and $\widetilde{P}_{00} = 1-\lambda/(\lambda+\mu+\varepsilon)$.

In the following we will derive bounds for the effect on the stationary distribution of the queue length in the M/M/1 queue when we increased the service rate.

For our bounds, we require bounds on the basic input entities such as π and T. In order to establish those bounds, we have to specify υ. Specifically, for $\beta > 1$, we will choose

$$\upsilon_{\beta}(s) = \beta^{s}, \qquad s \in S,$$

as our norm-defining mapping.

For ease of reference, we introduce the following condition:

(C)

$$\rho < 1 \quad \textit{and} \quad 1 < \beta < \frac{1}{\rho}.$$

Essential for our numerical bound on the deviation between stationary distributions π and $\widetilde{\pi}$ is a bound on the deviation of the transition kernel $\widetilde{P}$ from P. This bound is provided in the following lemma.

Lemma 0.1. *If condition* **(C)** *is satisfied, then*

$$\|\widetilde{P}-P\|_{\upsilon_\beta} \leq \frac{\lambda(1+\beta)\varepsilon}{(\lambda+\mu)(\lambda+\mu+\varepsilon)} = \Delta(\beta).$$

Proof. By definition, we have

$$\|\widetilde{P}-P\|_{\upsilon_\beta} = \sup_{k\geq 0} \frac{1}{\upsilon(k)} \sum_{j\geq 0} \upsilon(j)|\widetilde{P}_{kj}-P_{kj}|.$$

For $k=0$:

$$\begin{aligned}\Delta_0 &= \sum_{j\geq 0} \upsilon(j)|\widetilde{P}_{kj}-P_{kj}| \\ &= \upsilon(0)|\widetilde{P}_{00}-P_{00}| + \upsilon(1)|\widetilde{P}_{01}-P_{01}| \\ &= \frac{\lambda(1+\beta)\varepsilon}{(\lambda+\mu)(\lambda+\mu+\varepsilon)}.\end{aligned}$$

For $k \geq 1$:

$$\begin{aligned}\Delta_1 &= \sup_{k\geq 1} \frac{1}{\beta^k} \sum_{j\geq 0} \beta^j |\widetilde{P}_{kj}-P_{kj}| \\ &= \sup_{k\geq 1} \frac{1}{\beta^k} \left\{ \beta^{k+1} \left| \frac{\lambda}{\lambda+\mu+\varepsilon} - \frac{\lambda}{\lambda+\mu} \right| + \beta^{k-1} \left| \frac{\mu+\varepsilon}{\lambda+\mu+\varepsilon} - \frac{\mu}{\lambda+\mu} \right| \right\} \\ &= \frac{\varepsilon}{(\lambda+\mu)(\lambda+\mu+\varepsilon)} \left\{ \lambda\beta + \frac{\lambda}{\beta} \right\}\end{aligned}$$

Note that $\|\widetilde{P}-P\|_{\upsilon_\beta} = \max\{\Delta_0, \Delta_1\}$, and from $\Delta_1 < \Delta_0$, it follows that

$$\|\widetilde{P}-P\|_{\upsilon_\beta} \leq \Delta_0 = \Delta(\beta).$$

In the following lemma we will identify the range for β that leads to finite υ_β-norm of π.

Lemma 0.2. *Provided that* **(C)** *holds, the υ_β-norm of π is bounded by*

$$\|\pi\|_{\upsilon_\beta} = \frac{1-\rho}{1-\rho\beta} = c_0(\beta) < \infty.$$

Proof. The stationary distribution of P is known to be equal to $\pi(i) = \rho^i(1-\rho)$. Hence,

$$\|\pi\|_{\upsilon_\beta} = (1-\rho)\sum_{i=0}^{\infty} \beta^i \rho^i,$$

which is finite if $\beta\rho < 1$.

Note that $\rho < 1$ is assumed for stability and $\beta > 1$ is assumed in the definition of υ_β.

Lemma 0.3. *If condition* (**C**) *holds, then*

$$\|I-\Pi\|_{\upsilon} \leq \frac{2-\rho(\beta+1)}{1-\rho\beta} = c(\beta).$$

Proof. We have

$$\|I-\Pi\|_{\upsilon} \leq 1+\|\mathbf{1}\|_{\upsilon}\|\pi\|_{\upsilon}.$$

By definition, we obtain

$$\|\mathbf{1}\|_{\upsilon} = 1,$$

and $\|\mathbf{1}\|_{\upsilon}\|\pi\|_{v} = c_0(\beta)$. Therefore, $\|I-\Pi\|_{\upsilon} \leq 1+c_0(\beta) = c(\beta)$.

Let T denote the taboo Markov kernel for taboo state zero, that is, T is a deficient Markov kernel that avoids jumps to state zero; more specifically, for i, j let

$$T_{ij} = \begin{cases} 0, & \text{if } i=0, \\ P_{ij}, & \text{otherwise.} \end{cases} \tag{7}$$

Lemma 0.4. *Provided that condition* (**C**) *holds, it hold that*

$$\|T\|_{\upsilon_\beta} = \tau(\beta) < 1.$$

Proof. For $k=0$:

$$T\upsilon(0) = \sum_{j\geq 0} \upsilon(j)T_{0j} = \sum_{j\geq 0} \beta^j \times 0 = 0.$$

For $k \geq 1$:

$$\begin{aligned} T\upsilon(k) &= \sum_{j\geq 0} \upsilon(j)P_{kj} \\ &= \left\{\upsilon(k+1)\times\frac{\lambda}{\lambda+\mu}\right\} + \left\{\upsilon(k-1)\times\frac{\mu}{\lambda+\mu}\right\} \\ &= \beta^k\left\{\beta\frac{\lambda}{\lambda+\mu} + \frac{1}{\beta}\frac{\mu}{\lambda+\mu}\right\} && (8) \\ &= \tau(\beta)\times\upsilon(k), && (9) \end{aligned}$$

with $\tau(\beta) = (\lambda\beta/(\lambda+\mu)) + (\mu/\beta(\lambda+\mu))$.

Thus, $\|T\|_{\upsilon} = \tau(\beta)$, and it follows that the υ-norm of T is equal to $\tau(\beta)$. Provided that $\rho\beta < 1$, it holds that

$$\tau(\beta) = \frac{1+\rho\beta^2}{(1+\rho)\beta},$$

which yields $\tau(\beta) < 1$. To summarize,

$$\rho\beta < 1 \quad \Rightarrow \quad \|T\|_{\upsilon} = \tau(\beta) < 1, \text{for } \upsilon(k) = \beta^k.$$

which proves the claim.

Lemma 0.5. *Provided that condition* **(C)** *holds,* $\|P\|_{\upsilon_\beta}$ *is finite.*

Proof. Let H denote the matrix with row zero equal to P and all other rows equal to zero. Then,

$$T = P - H \Rightarrow P = T + H \Rightarrow \|P\|_{\upsilon_\beta} \leq \|T\|_{\upsilon_\beta} + \|H\|_{\upsilon_\beta}.$$

By Lemma 0.4, it holds that $\|T\|_{\upsilon_\beta} < \infty$ and for the proof of the claim it suffices to show that $\|H\|_{\upsilon_\beta} < \infty$, which follows from

$$\|H\|_{\upsilon_\beta} = \sum_{j \geq 0} \beta^j P_{0j} = \frac{\mu}{\lambda+\mu} + \beta \frac{\lambda}{\lambda+\mu} < \infty.$$

Let

$$B = \sup\{\beta > 1 : \tau(\beta) < 1\}.$$

Theorem 0.2. *Let $\rho < 1$. For $\beta \in B$, the deficient Markov kernel T defined in (7) is a residual of the Markov chain P with respect to υ_β for provided that $\tau(\beta) < 1$.*

Proof. Let

$$h(i) = \mathbf{1}_{i=0} = \begin{cases} 1, \text{ if } i = 0; \\ 0, \text{ otherwise,} \end{cases}$$

choose σ to be

$$\sigma_j = P_{0j} = \begin{cases} \frac{\mu}{\lambda+\mu}, \text{ if } j = 0; \\ \frac{\lambda}{\lambda+\mu}, \text{ if } j = 1; \\ 0, \quad \text{otherwise.} \end{cases}$$

In the following we will show that conditions a, b, c, and d hold for T defined in (7).

We start by verifying condition a and b. With the above definitions it holds that

$\pi h = \pi_0 = (1-\rho) > 0$, and
$\sigma \mathbf{1} = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} + 0 = 1$, here $\mathbf{1}$ is the vector having all the components equal to 1,
$\sigma h = \frac{\mu}{\lambda+\mu} + \left(0 \times \frac{\lambda}{\lambda+\mu}\right) + 0 = \frac{\mu}{\lambda+\mu} > 0$.

In the same way,

$$T_{ij} = P_{ij} - h(i)\sigma_j = \begin{cases} P_{0j} - \sigma_j = P_{0j} - P_{0j} = 0, \text{ if } i = 0; \\ P_{ij} - (0 \times \sigma_j) = P_{ij}, \qquad \text{otherwise,} \end{cases}$$

and from the definition of the kernel $(P_{ij})_{ij}$, it is obvious that $(T_{ij})_{ij} \geq 0$.

We not turn to condition c. In the following we will provide a sufficient condition such that $\|T\|_{\upsilon} < 1$ for $\upsilon(k) = \beta^k$, for all $k \geq 0$, with $\beta > 1$.

By Theorem 0.2, the general bound provided Theorem 0.1 can be applied to the Markov kernels P and $\widetilde{P}$ for our M/M/1 queue. Specifically, we will insert the individual bounds provided in Lemma 0.1, Lemma 0.2, Lemma 0.3 and Lemma 0.4, which yields the following result.

Theorem 0.3. *Let $\rho < 1$. For $\beta \in B$ such that*

$$\Delta(\beta) < \frac{1-\tau(\beta)}{c(\beta)},$$

it holds that

$$\|\widetilde{\pi} - \pi\|_{\upsilon_\beta} \leq \frac{c_0(\beta)c(\beta)\Delta(\beta)}{1-\tau(\beta)-c(\beta)\Delta(\beta)} = \mathbf{SSB}(\beta).$$

Proof. Note that $\beta \in B$ already implies $c_0(\beta) < \infty$ and $\tau(\beta) < 1$. Hence, Lemma 0.2 and Lemma 0.4 apply.

Following the line of thought put forward in Section 2, see (1), we will translate the norm bound in Theorem 0.3 to bounds for individual performance measures f.

Corollary 0.1. *Under the conditions put forward in Theorem 0.3, it holds for any f such that $\|f\|_{\upsilon_\beta} < \infty$ that*

$$|\widetilde{\pi} f - \pi f| \leq \|f\|_{\upsilon_\beta} \times \mathbf{SSB}(\beta) = h(\varepsilon, \beta).$$

Note that the bound in Corollary 0.1 has β as a free parameter. This give the opportunity to minimize the right hand side of the inequality in Corollary 0.1 with respect to β. For given ρ, this leads to the following optimization problem.

$$\min_{\beta \in B} h(\varepsilon, \beta)$$

$$\text{s.t.} \qquad \Delta(\beta) < \frac{1-\tau(\beta)}{c(\beta)}.$$

By inserting $\varepsilon > 0$ small, all inequalities can be made strict and in the above optimization problem can be solved using any standard technique.

4 Numerical Examples

In this section we will apply our bounds put forward in Theorem 0.3 and Corollary 0.1. For the numerical examples we set $\mu = 1$. We will discuss the following three cases in detail: the light traffic case $\rho = 0.1$, the medium traffic case $\rho = 0.6$, and the heavy traffic case $\rho = 0.9$. To illustrate the application of Corollary 0.1 to a particular performance function, we take $f(s) = s$ the identical mapping. In words, we are interested in the effect of perturbing the service rate by ε on the mean queue length. It is worth noting that in this case

$$\|f\|_{\upsilon_\beta} = \frac{1}{\ln(\beta)}\beta^{-\frac{1}{\ln(\beta)}}.$$

The light traffic case: We let $\rho = 0.1$. For applying our bounds we compute the value for β_{opt} that minimizes $h(\varepsilon,\beta)$. Then, we can compute the bounds put forward in Theorem 0.3 and Corollary 0.1 for various values for ε. The numerical results are presented in Table 1

Table 1 Errors comparative table for $\rho = 0.1$

ε	β_{opt}	$\|\pi - \tilde{\pi}\|_\beta$		$\lvert\pi f - \tilde{\pi} f\rvert$	
		Bound	True	Bound	True
0.01	2.6500	0.0021	$5.0481e-004$	$7.7964e-004$	$1.9056e-004$
0.1	2.6000	0.0204	0.0049	0.0079	0.0019
1	3	0.2641	0.0539	0.0884	0.0180

The medium traffic case: We let $\rho = 0.6$. In a similar manner, the mapping $h(\varepsilon,\beta)$ is minimized at β_{opt}. The perturbation $\varepsilon = 1$ is excluded for the medium traffic case. The numerical results are presented in Table 2, where the symbol "x" indicates that our bounds are not applicable.

Table 2 Errors comparative table for $\rho = 0.6$

ε	β_{opt}	$\|\pi - \tilde{\pi}\|_\beta$		$\lvert\pi f - \tilde{\pi} f\rvert$	
		Bound	True	Bound	True
0.01	1.2600	0.0945	0.0041	0.1504	0.0065
0.1	1.2000	1.6245	0.0294	3.2778	0.0594
1	x	x	x	x	x

The heavy traffic case: We let $\rho = 0.9$. The mapping $h(\varepsilon,\beta)$ is minimized at β_{opt}. Like for the medium traffic case, $\varepsilon = 1$ is excluded. The numerical results are presented in Table 3, where the symbol "x" indicates that our bounds are not applicable.

Table 3 Errors comparative table for $\rho = 0.9$

ε	β_{opt}	$\|\pi - \tilde{\pi}\|_\beta$		$\lvert\pi f - \tilde{\pi} f\rvert$	
		Bound	True	Bound	True
0.01	1.0480	0.2783	0.0021	2.1837	0.0166
0.1	1.0500	2.2340	0.0090	16.8448	0.0680
1	x	x	x	x	x

From these numerical results, it is easy to see that, the values of our bounds increase as the value of perturbation' parameter ε increases. Indeed, it is completely logical that the M/M/1 queue with service rate $\mu+\varepsilon$ is close to the M/M/1 queue with the same arrival flux and distribution of service time when ε tends to zero. Besides, we can notice the remarkable sensitivity of the Strong Stability Approach in the variation of the perturbation' parameter ε with regard to the value of the true distance. The bound obtained by the Strong Stability Approach is much more refined than that obtained by the true distance when the parameter ε tends to zero.

5 Concluding Remarks

The only input required from the perturbed queue is distance in υ-norm between P and $\widetilde{P}$. In this paper, we discussed an example, where P and $\widetilde{P}$ are Markov chains and a bound on $\|\widetilde{P}-P\|_\upsilon$ can be computed. On the other hand, as $\tilde{\pi}$ is known in closed form, one can compute $\|\tilde{\pi}-\pi\|_\upsilon$ directly and there is no imminent reason for applying our approach, other than the tutorial value of explaining the method for a simple example. As further research, we will show how to estimate $\|\widetilde{P}-P\|_\upsilon$ from a single sample path of the G/G/1 queue. This is topic of further research.

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Series Expansions in Queues with Server Vacation

Fazia Rahmoune and Djamil Aïssani

Abstract This paper provides series expansions of the stationary distribution of finite Markov chains. The work presented is a part of research project on numerical algorithms based on series expansions of Markov chains with finite state-space S. We are interested in the performance of a stochastic system when some of its parameters or characteristics are changed. This leads to an efficient numerical algorithm for computing the stationary distribution. Numerical examples are given to illustrate the performance of the algorithm, while numerical bounds are provided for quantities from some models like manufacturing systems to optimize the requirement policy or reliability models to optimize the preventive maintenance policy after modelling by vacation queuing systems.

1 Introduction

Let **P** denote the transition kernel of a Markov chain defined on a finite state-space S having unique stationary distribution π_P. Let **Q** denote the Markov transition kernel of the Markov chain modeling the alternative system and assume that **Q** has unique stationary distribution π_Q. The question about the effect of switching from **P** to **Q** on the stationary behavior is expressed by $\pi_P - \pi_Q$, the difference between the stationary distributions (Heidergott and Hordijk, 2003). In this work, we show that the performance measure of some stochastic models, which are gouverned by a finite Markov chain, can be obtained from other performance of more simple models, via series expansion method. Let $\| \, . \, \|_{tv}$ denote the total variation norm, then the above problem can be phrased as follows: Can $\| \pi_P - \pi_Q \|_{tv}$ be approximated or bounded

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in terms of $\| \mathbf{P}-\mathbf{Q} \|_{tv}$? This is known as *perturbation analysis of Markov chains (PAMC)* in the literature.

This paper is considered as a continuity of the work (Rahmoune and Aïssani, 2008), where quantitative estimate of performance measure has been established via strong stability method for some vacation queueing models. In this work, we will show that $\pi_P - \pi_Q$ can be arbitrarily closely approximated by a polynomial in $(\mathbf{Q}-\mathbf{P})D_P$, where D_P denotes the deviation matrix associated with $\mathbf{P}$. A precise definitions and notations will be given later. Starting point is the representation of π_Q given by:

$$\pi_Q = \sum_{n=0}^{k} \pi_P((\mathbf{Q}-\mathbf{P})D_P)^n + \pi_Q((\mathbf{Q}-\mathbf{P})D_P)^{k+1}; \tag{1}$$

for any $k \geq 0$. This series expansion of π_Q provides the means of approximating π_Q by $\mathbf{Q}$ and entities given via the $\mathbf{P}$ Markov chain only. We obtain a bound for the remainder term working with the weighted supremum norm, denoted by $\| \, . \, \|_v$, where v is some vector with positive non-zero elements, and for any $w \in \mathrm{R}^S$

$$\| w \|_v = \sup_{i\in S} \frac{w(i)}{v(i)}, \tag{2}$$

see, for example (Meyn and Tweedie, 1993). We will show that for our models

$$\pi_Q(s) - \left(\sum_{n=0}^{k} \pi_P((\mathbf{Q}-\mathbf{P})D_P)^n \right)(s) \leq d \, \| \, ((\mathbf{Q}-\mathbf{P})D_P) \, \|_v^{k+1}$$

for any $k \in \mathrm{N}$ and any $s \in \mathrm{S}$, where v can be any vector satisfying $v(s) \geq 1$ for $s \in \mathrm{S}$, and d is some finite computable constant. In particular, the above error bound can be computed without knowledge of π_Q.

The key idea of the approach is to solve for all k the optimization problem

$$\begin{cases} \min \| \, ((\mathbf{Q}-\mathbf{P})D_P)^k \, \|_v, & \text{subject to} \\ v(s) \geq 1, & \text{for } s \in \mathrm{S}. \end{cases} \tag{3}$$

The vector $v^\star$ thus yields the optimal measure of the rate of convergence of the series in (1). Moreover, the series in (1) tends to converge extremely fast which is due to the fact that in many examples $v^\star$ be found such that $\| \, ((\mathbf{Q}-\mathbf{P})D_P)^k \, \|_{v^\star} << 1$. The limit of the series (1) first appeared in (Cao, 1998), however, neither upper bounds for the remainder term nor numerical examples were given there. The derivation of this has been done in (Heidergott and Hordijk, 2003), which is a generalization of (Cao, 1998). The use of series expansion for computational purposes is not new. It has been used in the field of linear algebra (Cho and Meyer, 2001).
The work presented in this paper is part of research project on numerical algorithms based on series expansions of Markov chains as it was in Heidergott and Hordijk

(2003). The present paper establishes the main theoretical results. In particular, numerical examples are provided for vacation queueing systems.

2 Preliminaries on Finite Markov chains

Let S denote a finite set $\{1,\cdots,S\}$, with $0 < S < \infty$ elements. We consider Markov kernels on state space S, where the Markov kernel $\mathbf{P}^n$ is simply obtained from taking the nth power of $\mathbf{P}$. Provided it exists, we denote the unique stationary distribution of $\mathbf{P}$ by π_P and its ergodic projector by Π_P. For simplicity, we identify π_P and π_Q with Π_P and Π_P, respectively. Throughout the paper, we assume that $\mathbf{P}$ is aperiodic and unichain, which means that there is one closed irreducible set of states and a set of transient states. Let $|A|(i;j)$ denote the $(i;j)$th element of the matrix of absolute values of $A \in \mathrm{R}^{S\times S}$, and additionally we use the notation $|A|$ for the matrix of absolute values of A.

The main tool for this analysis is the v-norm, as defined in (2). For a matrix $A \in \mathrm{R}^{S\times S}$ the v-norm is given by

$$\| A \|_v \stackrel{def}{=} \sup_{i,\|w\|_v\leq 1} \frac{\sum_{j=1}^{S} |A(i,j)w(j)|}{v(i)}$$

Next we introduce v-geometric ergodicity of $\mathbf{P}$, see Meyn and Tweedie (1993) for details.

Definition 0.1. A Markov chain $\mathbf{P}$ is v-geometric ergodic if $c < \infty, \beta < 1$ and $N < \infty$ exist such that

$$\| \mathbf{P}^n - \Pi_P \|_v \leq c\beta^n, \quad \text{for all } \ n \geq N.$$

The following lemma shows that any finite-state aperiodic Markov chain is v-geometric ergodic.

Lemma 0.1. *For finite-state and aperiodic* $\mathbf{P}$ *a finite number N exists such that*

$$\| \mathbf{P}^n - \Pi_P \|_v \leq c\beta^n, \quad \textit{for all } \ n \geq N;$$

where $c < \infty$ *and* $\beta < 1$.

Proof. Because of the finite state space and aperiodicity.

3 Series Expansions in Queues with Server Vacation

We are interested in the performance of a queuing system with single vacation of the server when some of its parameters or characteristics are changed. The system as given is modeled as a Markov chain with kernel **P**, the changed system with kernel **Q**. We assume that both Markov chains have a common finite state space S. We assume too, as indicated earlier, that both Markov kernels are aperiodic and unichain. The goal of this section is to obtain the stationary distribution of **Q**, denoted by π_Q, via a series expansion in **P**. In the next section, we comment on the speed of convergence of this series. We summarize our results in an algorithm, presented in an other section. Finally, we illustrate our approach with numerical examples.

3.1 Description of the models

Let us consider the $M/G/1//N$ queueing systems with multiples vacations of the server modelling the reliability system with multiple preventives maintenances. We suppose that there is on the whole N machines in the system. Our system consists of a source and a waiting system (queue + service). Each machine is either in source or in the waiting system at any time. A machine in source arrives at the waiting system precisely, with the durations of inter-failure exponentially distributed with parameter λ) for repairment (corrective maintenance). The distribution of the service time is general with a distribution function $B(.)$ and mean b. The repairmen take maintenance period at each time the system is empty. If the server returns from maintenance finding the queue impty, he takes an other maintenance period (mulitiple maintenance). In addition, let us consider the $M/G/1//N$ queuing system with unique vacation of the server modelling the reliability system with periodic prenventives maintenances, having the same distributions of the inter-arrivals and the repair time previously described. In this model, the server (repairman) will wait until the end of the next activity period during which at least a customer will be served, before beginning another maintenance period. In other words, there is exactly only one maintenance at the end of each activity period at each time when the queue becomes empty *(exhaustive service)*. If the server returns from maintenane finding the queue nonempty, then the maintenance period finishes for beginning another activity period. We also suppose that the maintenance times V of the server are independent and iid, with general distribution function noted $V(x)$.

3.2 Transition Kernels

Let X_n (resp. $\bar{X}_n$) the imbedded Markov chains at the end moments of repair t_n for the n^{th} machine associated with the $M/G/1//N$ system with multiple maintenance (resp.to the system with the unique maintenance). In the same way, we define the following probabilities:

$$f_k = P[k \text{ broken down machines at the end of the preventive maintenance period}]$$

$$= C_N^k \int_0^{\infty} (1-e^{-\lambda t})^k e^{-(N-k)\lambda t} dV(t),\ k=\overline{0,N}. \quad (4)$$

for witch

$$\bar{\alpha}_k = \begin{cases} f_0 + f_1, & \text{for } k=1 \\ f_k, & \text{for } 2 \leq k \leq N. \end{cases}$$

and

$$\alpha_k = \frac{f_k}{1-f_0} \quad \text{for} \quad k=\overline{1,N}.$$

The one stage transition probabilities of the imbedded Markov chains X_n and $\bar{X}_n$ allow us to describe the general expression of the transition kernels $\mathbf{P} = (\mathbf{P}_{ij})_{ij}$ and $\mathbf{Q} = (\mathbf{Q}_{ij})_{ij}$ summarized below respectively.

$$\mathbf{P}_{ij} = \begin{cases} \sum\limits_{k=1}^{j+1} P_{j-k+1}\alpha_k, & \text{if} \quad i=0, j=\overline{0,N-1},\ k=\overline{1,N}, \\ P_{j-i+1} & \text{if} \quad 1 \leq i \leq j+1 \leq N-1, \\ 0 & \text{else.} \end{cases}$$

$$\mathbf{Q}_{ij} = \begin{cases} \sum\limits_{k=1}^{j+1} P_{j-k+1}\bar{\alpha}_k, & \text{if} \quad i=0, j=\overline{0,N-1},\ k=\overline{1,N}, \\ P_{j-i+1} & \text{if} \quad 1 \leq i \leq j+1 \leq N-1, \\ 0 & \text{else.} \end{cases}$$

Clearly, the Markov chain $\{\bar{X}_n\}_{n\in \mathrm{N}}$ is irreducible, aperiodic with finite state space $\mathrm{S} = \{0,1,\cdots,N-1\}$. So, we can applied the main theorical results established in this paper to this model, in order to approach another Markov chain whose transition kernel is neighborhood of its transition kernel $\mathbf{Q}$.

3.3 Series Development for π_Q

We write D_P for the deviation matrix associated with $\mathbf{P}$; in symbols:

$$D_P = \sum_{m=0}^{\infty} (\mathbf{P}^m - \Pi_P) \quad (5)$$

Note that D_P is finite for any aperiodic finite-state Markov chain. Moreover, the deviation can be rewritten as

$$D_P = \sum_{m=0}^{\infty} (\mathbf{P} - \Pi_P)^m - \Pi_P,$$

where $\sum\limits_{m=0}^{\infty} (\mathbf{P} - \Pi_P)^m$ is often referred to as the group inverse, see for instance Cao (1998) or Coolen-Schrijner and van Doorn (2002). A general definition which is valid for periodic Markov chain, can be found in, e.g., Puterman (1994).

Let $\mathbf{P}$ be unichain. Using the definition of D_P, we obtain:

$$(I-\mathbf{P})D_P = I - \Pi_P.$$

This is the Poisson equation in matrix format.

Let the following equation:

$$\Pi_Q = \Pi_P \sum_{n=0}^{k} ((\mathbf{Q}-\mathbf{P})D_P)^n + \Pi_Q((\mathbf{Q}-\mathbf{P})D_P)^{k+1}. \tag{6}$$

for $k \geq 0$, where:

$$H(k) \stackrel{def}{=} \Pi_P \sum_{n=0}^{k} ((\mathbf{Q}-\mathbf{P})D_P)^n,$$

is called a *series approximation of degree k* for $\Pi_Q, T(k)$, with

$$T(k) \stackrel{def}{=} \Pi_P((\mathbf{Q}-\mathbf{P})D_P)^k, \tag{7}$$

denotes the kth element of H(k), and

$$R(k) \stackrel{def}{=} \Pi_Q((\mathbf{Q}-\mathbf{P})D_P)^{k+1}, \tag{8}$$

is called the *remainder term* (see Heidergott et al, 2007, for details). The quality of the approximation provided by $H(k)$ is given through the remainder term $R(k)$.

3.4 Series Convergence

In this section we investigate the limiting behavior of $H(k)$ as k tends to ∞. We first establish sufficient conditions for the existence of the series.

Lemma 0.2. *(Heidergott and Hordijk, 2003) The following assertions are equivalent:*

(i) The series $\sum_{k=0}^{\infty} ((\mathbf{Q}-\mathbf{P})D_P)^k$ *is convergent.*
(ii) There are N *and* $\delta_N \in (0,1)$ *such that* $\| ((\mathbf{Q}-\mathbf{P})D_P)^N \|_v < \delta_N$.
(iii) There are κ *and* $\delta < 1$ *such that* $\| ((\mathbf{Q}-\mathbf{P})D_P)^k \|_v < \kappa\delta^k$ *for any* k.
(iv) There are N *and* $\delta \in (0;1)$ *such that* $\| ((\mathbf{Q}-\mathbf{P})D_P)^k \|_v < \delta^k$ *for any* $k \geq N$.

Proof. See Heidergott and Hordijk (2003).

The fact that the maximal eigenvalue of $|(\mathbf{Q}-\mathbf{P})D_P|$ is smaller than 1 is necessary for the convergence of the series $\sum_{k=0}^{\infty} ((\mathbf{Q}-\mathbf{P})D_P)^k$.

Remark 0.1. Existence of the limit of $H(k)$, see (i) in Lemma 0.3, is equivalent to an exponential decay in the v-norm of the elements of the series, see (iv) in Lemma 0.2. For practical purposes, one needs to identify the decay rate δ and the threshold value N after which the exponential decay occurs. The numerical experiments have shown that the condition (ii) in Lemma 0.2 is the most convenient to work with. More specifically, we work with the following condition (C) as in Heidergott and Hordijk (2003), which is similar to the geometric series convergence criterion.

The Condition(C)**:** There exists a finite number N such that we can find $\delta_N \in (0;1)$ which satisfies:

$$\| ((\mathbf{Q}-\mathbf{P})D_P)^N \|_v < \delta^N;$$

and we set

$$c^v_{\delta_N} \stackrel{def}{=} \frac{1}{1-\delta_N} \| \sum_{k=0}^{N-1} ((\mathbf{Q}-\mathbf{P})D_P)^k \|_v$$

As shown in the following lemma, the factor $c^v_{\delta_N}$ in condition (C) allows to establish an upper bound for the remainder term that is independent of Π_Q.

Lemma 0.3. *Under (C) it holds that:*

(i) $\| R(k-1) \|_v \leq c^v_{\delta_N} \| T(k) \|_v$ *for all* k,

(ii) $\lim\limits_{k\to\infty} H(k) = \Pi_P \sum\limits_{n=0}^{\infty} ((\mathbf{Q}-\mathbf{P})D_P)^n = \Pi_Q$

Proof. To proof the lemma it is sufficient to use the definition of the norm $\| . \|_v$ and the remainder term $R(k-1)$, using the condition (iv) of Lemma 0.2.

Remark 0.2. An example where the series $H(k)$ fails to converge is illustrated in Heidergott and Hordijk (2003).

Remark 0.3. The series expansion for Π_Q put forward in the assertion (ii) in Lemma 1 is well known; see Cao (1998) and Kirkland (2003) for the case of finite Markov chains and Heidergott and Hordijk (2003) for the general case. It is however worth noting that in the aforementioned papers, the series was obtained via a differentiation approach, whereas the representation is derived in this paper from the elementary equation 6.

Remark 0.4. Provided that $\det(I-(\mathbf{Q}-\mathbf{P})D_P) \neq 0$, one can obtain π_Q from

$$\pi_Q = \Pi_P(I-(\mathbf{Q}-\mathbf{P})D_P)^{-1} \tag{9}$$

Moreover, provided that the limit

$$\lim_{k\to\infty} H(k) = \lim_{k\to\infty} \pi_P \sum_{n=0}^{\infty} ((\mathbf{Q}-\mathbf{P})D_P)^n$$

exists (see Lemma 0.3 for sufficient conditions), it yields π_Q as $\pi_P \sum_{n=0}^{\infty}((\mathbf{Q}-\mathbf{P})D_P)^n$.

Remark 0.5. Note that a sufficient condition for (C) is

$$\| (\mathbf{Q}-\mathbf{P})D_P \|_v < \delta, \;\; \delta < 1. \tag{10}$$

In Altman et al (2004); Cho and Meyer (2001) it is even assumed that

$$\| (\mathbf{Q}-\mathbf{P})D_P \|_v < g_1, \tag{11}$$

with $g_1 > 0$ a finite constant, and

$$\| D_P \|_v < \frac{c}{1-\beta}, \tag{12}$$

with $c > 0$ and $0 < \beta < 1$ finite constants. If

$$\frac{g_1 c}{1-\beta} < 1, \tag{13}$$

then (10) and hence (C) is clearly fulfilled. Hence, for numerical purposes these conditions are too strong.

3.5 The remainder term Bounds

The quality of approximation by $H(k-1)$ is given by the remainder term $R(k-1)$ and in applications v should be chosen such that it minimizes $c^v_{\delta_N} \| T(k) \|_v$, thus minimizing our upper bound for the remainder term. For finding an optimal upper bound, since $c^v_{\delta_N}$ is independent of k, we focus on $T(k)$. Specifically, we have to find a bounding vector v that minimizes $\| T(k) \|_v$ uniformly w.r.t. k. As the following theorem shows, the unit vector, denoted by 1, with all components equal to one, yields the minimal value for $\| T(k) \|_v$ for any k.

Theorem 0.1. *(Heidergott and Hordijk, 2003) The unit vector* 1 *minimizes* $\| T(k) \|_v$ *uniformly over k, i.e.,*

$$\forall k \geq 1 : inf_v \| T(k) \|_v = \| T(k) \|_1 \tag{14}$$

Remark 0.6. It can be shown as for the results in Altman et al (2004) and Cho and Meyer (2001), that the smallest $\frac{cg_1}{1-\beta}$ is precisely the maximal eigenvalue of $|D_P|$. Again we note that often the product of these maximal eigenvalues is not smaller than 1. If this is the case, then according to Altman et al (2004) and Cho and Meyer (2001) we cannot decide whether the series $H(k)$ converges to Π_Q. Hence, their condition is too restrictive for numerical purposes.

3.6 Algorithm

In this section we describe a numerical approach to computing our upper bound for the remainder term $R(k)$. We search for N such that $1 > \delta_N \stackrel{def}{=} \| ((\mathbf{Q}-\mathbf{P})D_P)^N \|_1$, which implies that for N and δ_N, the condition (C) holds. Then the upper bound for $R(k)$ is obtained from $c^1_{\delta_N} \| ((\mathbf{Q}-\mathbf{P})D_P)^{k+1} \|_1$. Based on the above, the algorithm that yields an approximation for π_Q with ε precision can be described, with two main parts. First $c^1_{\delta_N}$ is computed. Then, the series can be computed in an iterative way until a predefined level of precision is reached.

The Algorithm

Chose precision $\varepsilon > 0$. Set $k = 1, T(1) = \Pi_P(\mathbf{Q}-\mathbf{P})D_P$ and $H(0) = \Pi_P$.

Step 1: Find N such that $\| ((\mathbf{Q}-\mathbf{P})D_P)^N \|_1 < 1$. Set $\delta_N = \| ((\mathbf{Q}-\mathbf{P})D_P)^N \|_1$ and compute

$$c^1_{\delta_N} = \frac{1}{1-\delta_N} \| \sum_{k=0}^{N-1} ((\mathbf{Q}-\mathbf{P})D_P)^k \|_1 .$$

Step 2: If

$$c_\delta \| T(k) \|_1 < \varepsilon,$$

the algorithm terminates and $H(k-1)$ yields the desired approximation. Otherwise, go to step 3.

Step 3: Set $H(k) = H(k-1) + T(k)$. Set $k := k+1$ and $T(k) = T(k-1)(\mathbf{Q}-\mathbf{P})D_P$. Go to step 2.

Remark 0.7. Algorithm 1 terminates in a finite number of steps, since $\sum_{k=0}^{\infty} ((\mathbf{Q}-\mathbf{P})D_P)^k$ is finite, .

3.7 Numerical Application

The present paper established the main theoretical results, and the analysis provided applies to the case of optimization of preventive maintenance in reparable reliability models. The application of this algorithm step by step gives us the following results.

This part of the paper is reserved for theoretical and numerical results obtained via series expansion method to obtain the development of the stationary distribution of the $M/G/1//N$ queueing models with single server vacation, witch modeless reliability system with preventive maintenance.

Let S the state space of the imbedded Markov chains X_n and $\bar{X}_n$ of the both considered queueing systems. Note that the both chains are irreducible and aperiodic, with finite state space S, so they are v-geometric ergodic. We note by D_P the deviation matrix associated to $\bar{X}_n$ chain, and by π_P its stationary distribution, with stationary projector Π_P. In the same time, π_Q is the stationary distribution of X_n, with the projector Π_Q.

We want to express π_Q in terms of puissance series on $(P-Q)D_P$ and π_P as follows:

$$\pi_Q = \sum_{n=0}^{\infty} \pi_P((Q-P)D_P)^n; \tag{15}$$

We show that this series is convergent. In fact, since the state space of the both chains is finite, so we can give the first following elementary result:

Lemma 0.4. *Let X_n and $\bar{X}_n$ the imbedded Markov chains of the $M/G/1//N$ queueing system with server vacation and the classical $M/G/1//N$system respectively. Then, the finite number N exist and verified the following:*

$$\| P^n - \Pi_P \|_v \leq c\beta^n, \ \textit{for all} \ \ n \geq N; \tag{16}$$

where $c < \infty$, $\beta < 1$.

For the same precedent raisons we give the most important result about the deviation matrix D_P associated to the imbedded Markov chain $\bar{X}_n$.

Lemma 0.5. *Let $\bar{X}_n$ the imbedded Markov of the classical $M/G/1//N$ queueing system and D_P its deviation matrix. Then, D_P is finite.*

Using Lemma 0.2, we obtain the following result about the required series expansion:

Lemma 0.6. *Let π_P (resp. π_Q) the stationary distribution of the $M/G/1//N$ classical system, (resp. $M/G/1//N$ system with unique vacation), and D_P the associated deviation matrix. Then, the series*

$$\sum_{n=0}^{\infty} \pi_P((Q-P)D_P)^n; \tag{17}$$

converge normally then uniformly.

This result is equivalent to say that the reminder term $R(k)$ is uniformly convergent to zero.

From the condition (C) and the Lemma 0.3, the sum function of the series 15 is the stationary vector π_Q.

Lemma 0.7. *Let π_P (resp. π_Q) the stationary distribution of the $M/G/1//N$ classical system, (resp. $M/G/1//N$ with vacation of the server), and D_P the associated deviation matrix. Then, the series*

$$\pi_Q = \sum_{n=0}^{\infty} \pi_P((Q-P)D_P)^n; \tag{18}$$

converge uniformly to the stationary vector π_Q.

From the work of Heidergot, we describe in this section a numerical approach to compute the supremum borne of the reminder term $R(k)$. We ask about the number N as:

$$\delta_N = \| ((Q-P)D_P)^N \|_1 < 1,$$

witch implies that the condition (C) is verified for N and δ_n. Then the limit of $R(k)$ is obtained from:

$$\| ((Q-P)D_P)^{k+1} \|_1 < c^1_{\delta_N}.$$

The performance measure for witch we are interesting is the mean number of costumers at the stationary state in the system.

The considered entries parameters are: $\bar{N} = 5$, $\lambda = 2$, service rate $\rightarrow Exp(\mu_s = 5)$, vacation rate $\rightarrow Exp(\mu_v = 300)$.

Our goal is to compute approximatively the quantities $\pi^\star w$.

The error to predict the stationary queue length via the quantities $H(n)$ is then given and illustrated in the Figure1.

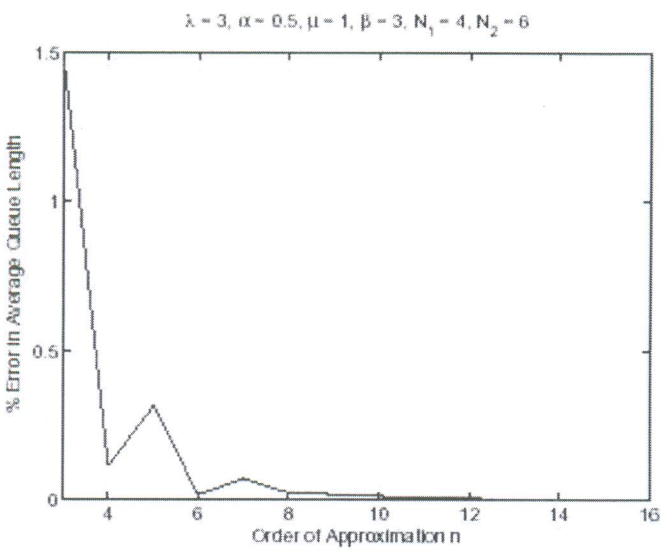

Fig. 1 Error in Average queue length

The figure show that

$$|\frac{\pi^\star w - H(n)w}{\pi^\star w}| \tag{19}$$

is a graph on n. The numerical value of πw is 2.4956. For this example, we have obtained $N = 14, \delta_N = 0.9197$ and $c^1_{\delta_N} = 201.2313$.

The algorithm terminates when the upper bound for $\| R \|_1$ given by $c^1_{\delta_N} \| R \|_1$ is under the value ε. By taking $\varepsilon = 10^{-2}$, the algorithm compute $\pi^\star w$ juste to the precision $10^{-2} \| w \|_1$.

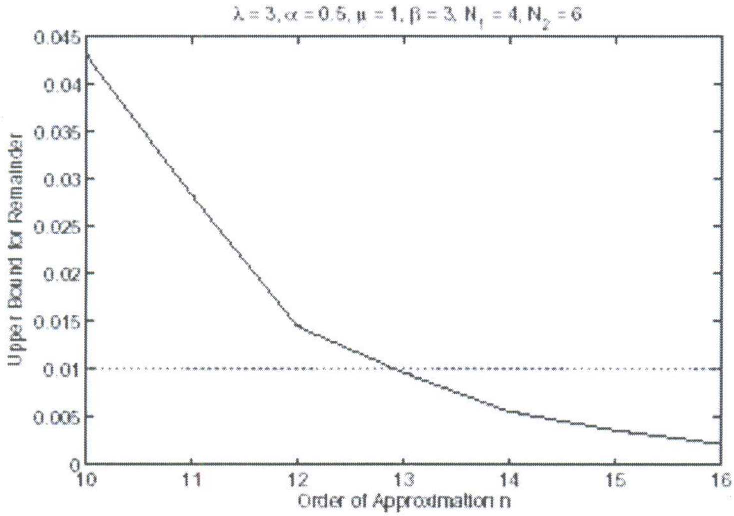

Fig. 2 Relative error of the upper bound of the remainder term

From this figure we conclude that $\pi \sum_{k=0}^{13} ((Q^\star - Q)D)^k w$ approximates $\pi^\star w$ with a maximal absolute error $\varepsilon \| w \|_1 = 3 * 10^{-2}$.

4 Conclusion

In this work, we have presented a part of research project on numerical algorithms based on series expansions of finite Markov chains. We are interested in the performance of a stochastic system when some of its parameters or characteristics are perturbed. This leads to an efficient numerical algorithm for computing the stationary distribution. We have shown theoretically and numerically that introducing a small disturbance on the structure of maintenance policy in $M/G/1//N$ system with multiples maintenances after modelling by queues with server vacation, we obtain the $M/G/1//N$ system with single maintenance policy (periodic maintenance). Then characteristics of this system can be approximated by those of the $M/G/1//N$ system with periodic maintenance, with a precision which depends on the disturbance, in other words on the maintenance parameter value.

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Contents

Appendix A
International Scientific Board

The chair of the international scientific board of the 2nd rapid modelling conference "Rapid Modelling and Quick Response: Intersection of Theory and Practice" consisted of:

- Gerald Reiner (University of Neuchâtel, Switzerland)

  Members of the international scientific board as well as referees are:

- Djamil Aïssani (LAMOS, University of Béjaia, Algeria)
- Michel Bierlaire (EPFL, Switzerland)
- Cecil Bozarth (North Carolina State University, USA)
- Bénamar Chouaf (University of Sidi Bel Abes, Algeria)
- Lawrence Corbett (Victoria University of Wellington, New Zealand)
- Krisztina Demeter (Corvinus University of Budapest, Hungary)
- Suzanne de Treville (University of Lausanne, Switzerland)
- Barb Flynn (Indiana University, USA)
- Gerard Gaalman (University of Groningen, The Netherlands)
- Ari-Pekka Hameri (University of Lausanne, Switzerland)
- Petri Helo (University of Vaasa, Finland)
- Olli-Pekka Hilmola (Lappeenranta University of Technology, Finland)
- Werner Jammernegg (Vienna University of Economics and Business Administration, Austria)
- Matteo Kalchschmidt (University of Bergamo, Italy)
- Ananth Krishnamurthy (University of Wisconsin-Madison, USA)
- Doug Love (Aston Business School, UK)
- Jose Antonio Dominguez Machuca (University of Sevilla, Spain)
- Carolina Osorio (EPFL, Switzerland)
- Jeffrey S. Petty (Lancer Callon, UK)
- Reinhold Schodl (University of Neuchâtel, Switzerland)
- Boualem Rabta (University of Neuchâtel, Switzerland)
- Nico J. Vandaele (Catholic University Leuven, Belgium)

Preface

Rapid Modelling and Quick Response - Intersection of Theory and Practice

This volume is a sequel of the 1st Rapid Modelling Conference proceedings volume that focused on Rapid Modelling for increasing competitiveness. The main focus of the 2nd Rapid Modelling Conference proceedings volume "Rapid Modelling and Quick Response - Intersection of Theory and Practice" is the transfer of knowledge from theory to practice, providing the theoretical foundations for successful performance improvement (based on lead time reduction, etc. as well as financial performance measures). Furthermore illustrations will be given by teaching/business cases as well as success stories on new software tools in this field as well as new approaches. In general, Rapid Modelling is based on queueing theory but other mathematical modelling techniques as well as simulation models which facilitate the transfer of knowledge from theory to application are of interest as well.

Together with the proceedings volume of selected papers presented a the 1st Rapid Modelling Conference "Increasing Competitiveness - Tools and Mindset" the interested reader should have a good overview on what is going on in this field. The objective of this conference series is to provide an international, multidisciplinary platform for researchers and practitioners to create and exchange knowledge on increasing competitiveness through Rapid Modelling. In this volume, we demonstrate that lead time reduction (through techniques ranging from quick response manufacturing to lean production) is very important but not enough. Additional factors such as risk, costs, revenues, environment, etc. have to be considered as well. We accepted papers that contribute to these themes in the form of:

- Rapid Modelling
- Case study research, survey research, action research, longitudinal research
- Theoretical papers
- Teaching/business case studies

Relevant topics are:

- Queueing Theory
- Rapid Modelling in Manufacturing and Logistics
- Rapid Modelling in Services
- Rapid Modelling and Financial Performance Measurement
- Product and Process Development
- Supply Chain Management

Based on these categories, the proceedings volume has been divided into six chapters and brings together selected papers which present different aspects of the 2nd Rapid Modelling Conference. These papers are allocated based on their main contribution. All papers passed through a double-blind referee process to ensure their quality.

While the RMC10 (2nd Rapid Modelling Conference "Rapid Modelling and Quick Response - Intersection of Theory and Practice") takes place at the University of Neuchâtel, located in the heart of the city of Neuchâtel, Switzerland, it is based on a collaboration with the project partners within our IAPP Project (No. 217891, see also *http://www.unine.ch/iene-kje*). We are happy to have brought together authors from Algeria, Austria, Belgium, United Kingdom, Finland, Germany, Hungary, Italy, Sweden, Switzerland, Turkey and the United States of America.

Acknowledgement

We would like to thank all those who contributed to the conference and this proceedings volume. First, we wish to thank all authors and presenters for their contribution. Furthermore, we appreciate the valuable help from the members of the international scientific board, the referees and our sponsors (see the Appendix for the appropriate lists).

In particular, our gratitude goes to our team at Enterprise Institute at the University of Neuchâtel, Gina Fiore Walder, Reinhold Schodl, Boualem Rabta, Arda Alp, Gil Gomes dos Santos, Yvan Nieto, who supported this conference project and handled the majority of the text reviews as well as the formating work with LaTex. Ronald Kurz created the logo of our conference and he took over the development of the conference homepage *http://www.unine.ch/rmc10*.

Finally, it has to be mentioned that the conference as well as the book are supported by the EU SEVENTH FRAMEWORK PROGRAMME - THE PEOPLE PROGRAMME - Industry-Academia Partnerships and Pathways Project (No. 217891) "How revolutionary queuing based modelling software helps keeping jobs in Europe. The creation of a lead time reduction software that increases industry competitiveness and supports academic research."

Neuchâtel, June 2010 *Gerald Reiner*

Optimal management of equipments of the BMT Containers Terminal (Bejaia's Harbor)

D. Aïssani, M. Cherfaoui , S. Adjabi, S. Hocine and N. Zareb

Abstract The BMT (Bejaia Mediterranean Terminal) Company of Bejaia's harbor became aware that the performances of the terminal with containers is measured by the time of stopover, the speed of the operations, the quality of the service and the cost of container's transit. For this end, the company has devoted several studies to analyze the performance of its terminal: elaboration of a global model for the "loaded / unloaded" process, modeling of the system by another approach which consists of the decomposition of the system into four independent subsystems (namely: the "loading" process, the "unloading" process, the "full-stock" process and the "empty-stock" process - (see Aïssani et al, 2009; Aïssani et al, 2009). The models used in this last study describe in detail the comportment of the real systems and the obtained results given by the simulators corresponding to each model are approximately the same as the real values. It is the reason for which the company wants to exploit these models in order to determine an optimal management of its equipments.

Perturbation Analysis of M/M/1 Queue

Karim Abbas and Djamil Aïssani

Abstract This paper treats the problem of evaluating the sensitivity of performance measures to changes in system parameters for a specific class of stochastic models. Motivated by the problem of the coexistence on transmission links of telecommunication networks of elastic and unresponsive traffic, we study in this paper the impact on the stationary characteristics of an M/M/1 queue of a small perturbation in the server rate. For this model we obtain a new perturbation bound by using the Strong Stability Approach. Our analysis is based on bounding the distance of stationary distributions in a suitable functional space.

Series Expansions in Queues with Server Vacation

Fazia Rahmoune and Djamil Aïssani

Abstract This paper provides series expansions of the stationary distribution of finite Markov chains. The work presented is a part of research project on numerical algorithms based on series expansions of Markov chains with finite state-space S. We are interested in the performance of a stochastic system when some of its parameters or characteristics are changed. This leads to an efficient numerical algorithm for computing the stationary distribution. Numerical examples are given to illustrate the performance of the algorithm, while numerical bounds are provided for quantities from some models like manufacturing systems to optimize the requirement policy or reliability models to optimize the preventive maintenance policy after modelling by vacation queuing systems.

Gerald Reiner
Editor
Rapid Modelling and Quick Response
Intersection of Theory and Practice

Rapid Modelling and Quick Response is a proceedings volume of selected papers presented at the Second Rapid Modelling Conference "Quick Response – Intersection of Theory and Practice". It presents new research developments, as well as business/teaching cases in the field of rapid modelling and quick response linked with performance improvements (based on lead time reduction, etc., as well as financial performance measures). This volume is a sequel to the First Rapid Modelling Conference proceedings volume that focused on rapid modelling for increasing competitiveness. The main focus of this second proceedings volume is the transfer of knowledge from theory to practice, providing the theoretical foundations for successful performance improvement. Furthermore, illustrations are given by teaching/business cases, as well as by success stories about new software tools in this field, and new approaches.

The interested reader (researcher, as well as practitioner) will gain a good overview on new developments in this field. This conference volume is a must-have for innovative production managers, as well as for managers of service-providing processes. The theoretical, as well as the empirical/practical, pieces of work presented will change the mindset of the interested reader. *Rapid Modelling and Quick Response* will also contribute to the scientific communities of operations management, production management, supply chain management, industrial engineering and operations research. This volume and the presented research work, teaching/business cases, as well as software tools, can also be used for the education of students and executive managers.

Rapid Modelling and Quick Response is supported by the EU Seventh Framework Programme — The People Programme — Industry-Academia Partnerships and Pathways Project (No. 217891) "How revolutionary queuing based modelling software helps keeping jobs in Europe. The creation of a lead time reduction software that increases industry competitiveness and supports academic research."

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