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# Performance Evaluation in a Queueing System $M_2/G/1$

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**Abstract.** In this communication, we use the strong stability method to approximate the characteristics of the  $M_2/G/1$  queue with preemptive resume priority by those of the  $M/G/1$  one. For this, we first prove the stability fact and next obtain quantitative stability estimates with an exact computation of constants.

**Keywords:** Strong stability, Approximation, Preemptive priority, Markov chain.

## Stability of Two-Stage Queues with Blocking

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**Abstract.** Queueing networks are known to provide a useful modeling and evaluation tool in computer and telecommunications. Unfortunately, realistic features like finite capacities, retrials, priority, ... usually complicate or prohibit analytic solutions. Numerical and approximate computations as well as simplifications and performance bounds for queueing networks therefore become of practical interest. However, it is indispensable to delimit the stability domain wherever these approximations are justified.

In this paper we applied for the first time the strong stability method to analyze the stability of the tandem queues  $[M/G/1 \rightarrow .M/1/1]$ . This enables us to determine the conditions for which the characteristics of the network with retrials  $[M/G/1/1 \rightarrow .M/1/1]$ , can be approximated by the characteristics of the ordinary network  $[M/G/1 \rightarrow .M/1/1]$  (without retrials).

**Keywords:** Queueing networks, tandem queues, Stability, Retrials, Blocking, Markov chain.



Hoai An Le Thi  
Pascal Bouvry  
Tao Pham Dinh (Eds.)

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# Performance Evaluation in a Queueing System $M_2/G/1$

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**Abstract.** In this communication, we use the strong stability method to approximate the characteristics of the  $M_2/G/1$  queue with preemptive resume priority by those of the  $M/G/1$  one. For this, we first prove the stability fact and next obtain quantitative stability estimates with an exact computation of constants.

**Keywords:** Strong stability, Approximation, Preemptive priority, Markov chain.

## 1 Introduction

Queueing phenomena occur in several real situations when resources can not immediately render the current or the kind of service required by their users. The theory of queues is particularly well adapted to the study of the performance of computer systems and communication networks. Such systems often trait classes of request of different priorities. Nowadays, the introduction of priorities in such systems is frequent and often motivated by the need to increase their performance and quality of service. A complete presentation of the area was the subject of monographs of N.K.Jaiswal [7] or of B.V.Gneedenko and al [5]. Nevertheless, these non markovien systems are complex then difficult to study and their characteristic are obtained in a very complicated way of the parameters of the system [7].

In addition, the including some complex systems in queueing networks, dos not allow the use of the existing queueing networks models (with product form solutions). This why it is interesting to study the proximity of characteristics of some complex systems by those of the simpler and more exploitable one. The purpose of this paper is to obtain the conditions and estimations of strong stability of an imbedded Markov chain in an  $M_2/G/1$  system with a preemptive priority. This is to approximate this system by the  $M/G/1$  model. Indeed, the characteristic of the queue  $M/G/1$  are obtained in an explicit form and this last allows the use of the product form solutions.

In the stability theory, we establish the domain within a model may be used as a good approximation or idealization to the real system under consideration. The stability methods allow to investigate qualitative proprieties of the

system, in particular its robustness. In addition, using these approach, bounds can be obtained in an explicit form and approximations can be made rigourously [13]. Indeed, measure of robustness of the system also need to be evaluated in comparison to the often studied measures of performance and efficiency [11]. The first results on the stability have been obtained by Rosseberg [12], Gnedenko [5], Franken [4] and Kennedy [10]. Afterwards, several papers have considered various situations and various approaches. Stoyen proposed the weak convergence method [13] used to investigate proprieties of stability of homogeneous Markov processus. Kalashinkov and Tsitsiashvili proposed the method of test functions [8] which consist in constructing a test function allowing to compare the behavior of the perturbed system (real model) with the non-perturbed system (ideal model). Borovkov proposed the renewal method [3] whose advantage comes from the fact that it allows to obtain theorems of ergodicity and stability with minimal conditions. Zolotariev and Rachev proposed the metric method [14], [11]. Ipsen and Meyer proposed the uniform stability method [6] whose aim to analyze the sensitivity of individual stationary probabilities to perturbations in the transition probabilities of finite irreducible Markov chains. Kartashov and Aïssani proposed the strong stability method [1]. In contrast to other methods, this technique suppose that the perturbations of the transition kernel (associated to the Markov chain describing the system) is small with respect to a certain norm. Such strict conditions allows us to obtain better estimations on the characteristics of the perturbed chain. This article is struted as follow. In the section 2, we clarify the Markov chains and their transition operation describing the system and the basic theorems of the strong stability method. In the section 3, we prove the strong stability in an  $M/G/1$  queue. On other words we clarify the condition under which the  $M/G/1$  system can be approximate the  $M_2/G/1$  system with priority. The section 4 gives the error of approximation on the stationary distribution of the number of request when the intensity of the flux is sufficing small.

## 2 Description of the Systems

Let us consider a queueing system  $M_2/G/1$  with preemptive priority. Priority and non priority request arrive at service mechanism in poisson streams with mean rates  $\lambda_1$  and  $\lambda_2$  respectively. The service of priority and non-priority request are distributed with probability density  $b(t)$ . The service of a non-priority request may be interrupted by the arrival of a priority request. When the later completes its service, the interrupted begin again its service if no priority request are waiting. The service time of the non priority request up to the interruption is distributed with probability density  $b^*(t)$ . The state of the  $M_2/G/1$  queueing system with preemptive priority at time  $t$  can be described by using the method of imbedded Markov chain, for this we define:

$X_{n+1}^i$ : the number of priority request(respectively non-priority request) in the system at instant  $t_{n+1}$ .

If  $X_n^1 \neq 0$ :  $t_{n+1}$  is the instant of "end of service" of priority request.

If  $X_n^1 = 0$ :  $t_{n+1}$  is the instant of end of service of non-priority request” or ”instant of interruption of priority request”

$A_{n+1}^i$ ,  $i = 1, 2$ : is a random variable that represents the number of the priority (respectively non priority) request arriving during the  $(n + 1)^{th}$  service.

- If  $t_{n+1}$  is the instant of the end of service of priority (respectively non priority) request, the distribution of  $A_{n+1}^i$ ,  $i = 1, 2$  is:

$$a_k^i = P(A_{n+1}^i = k) = \int_0^\infty \frac{(\lambda_i t)^k}{k!} e^{-\lambda_i t} b(t) dt, \quad i = 1, 2$$

-If  $t_{n+1}$  is the instant of interruption, the distribution of  $A_{n+1}^1$  is:

$$a_k^1 = P(A_{n+1}^1 = 1) = \int_0^\infty \lambda_1 t e^{-\lambda_1 t} b^*(t) dt.$$

The random variables  $A_{n+1}^1, A_{n+1}^2$  are independent between them.

**Lemma 1.** *The sequence  $(X_{n+1}^1, X_{n+1}^2)$  forms a Markov chain of transition operator  $P_{k,l}(i, j)_{i,j \geq 0}$  defined by:*

$$P_{k,l}(i, j) = \begin{cases} \bullet \int_0^\infty \frac{(\lambda_1 t)^{i-k+1}}{(i-k+1)!} \frac{(\lambda_2 t)^{j-l}}{(j-l)!} e^{-(\lambda_1 + \lambda_2)t} b(t) dt, \\ \text{if } k > 0, j \geq l, l \geq 0, i \geq k - 1. \\ \bullet \int_0^\infty e^{-\lambda_1 t} b(t) dt \int_0^\infty \frac{(\lambda_2 t)^{j-l+1}}{(j-l+1)!} e^{-\lambda_2 t} b(t) dt \\ + \int_0^\infty (\lambda_1 t) e^{-\lambda_1 t} b^*(t) dt \int_0^\infty \frac{(\lambda_2 t)^{j-l-1}}{(j-l-1)!} e^{-\lambda_2 t} b^*(t) dt. \\ \text{if } j \geq l - 1, k = 0, i = 0, l \neq 0. \\ \bullet \frac{\lambda_1}{\lambda_1 + \lambda_2} \int_0^\infty \frac{(\lambda_1 t)^i}{i!} \frac{(\lambda_2 t)^j}{j!} e^{-(\lambda_1 + \lambda_2)t} b(t) dt + \frac{\lambda_2}{\lambda_1 + \lambda_2} \times \\ \times [\int_0^\infty e^{-\lambda_1 t} b(t) dt \int_0^\infty \frac{(\lambda_2 t)^j}{j!} e^{-\lambda_2 t} b(t) dt \\ + \int_0^\infty (\lambda_1 t) e^{-\lambda_1 t} b^*(t) dt \int_0^\infty \frac{(\lambda_2 t)^j}{j!} e^{-\lambda_2 t} b^*(t) dt], \\ \text{if } i \geq 0, j \geq 0, k = 0, l = 0. \end{cases}$$

*Proof.* In order to calculate  $P_{k,l}(i, j)_{i,j}$  we consider the different cases:

**Case 1 :**  $X_n^1 \neq 0, X_n^2 \geq 0$

In this case  $t_{n+1}$  is the end of service of priority request, and the sequence

$$(X_{n+1}^1, X_{n+1}^2) \text{ is given by : } \begin{cases} X_{n+1}^1 = X_n^1 + A_{n+1}^1 - 1, \\ X_{n+1}^2 = X_n^2 + A_{n+1}^2. \end{cases}$$

The probability of transition  $P_{k,l}(i, j)$  is:

$$P_{k,l}(i, j) = P(A_{n+1}^1 = i - k + 1, A_{n+1}^2 = j - l) = \int_0^\infty \frac{(\lambda_1 t)^{i-k+1}}{(i-k+1)!} \frac{(\lambda_2 t)^{j-l}}{(j-l)!} e^{-(\lambda_1 + \lambda_2)t} b(t) dt$$

**Case 2 :**  $X_n^1 = 0, X_n^2 \neq 0$

If  $t_{n+1}$  is the “end of service of non-priority request”, the sequence

$$(X_{n+1}^1, X_{n+1}^2) \text{ is given by : } \begin{cases} X_{n+1}^1 = 0, \\ X_{n+1}^2 = X_n^2 + A_{n+1}^2 - 1. \end{cases}$$

That explains  $A$  : “ no priority request arrives during the service of the non-priority request”. The probability of  $A$  is :  $P(A) = \int_0^\infty e^{-\lambda_1 t} b(t) dt$ .

If  $t_{n+1}$  is the “ instant of interruption of priority request”, the sequence

$$(X_{n+1}^1, X_{n+1}^2) \text{ is given by: } \begin{cases} X_{n+1}^1 = 1, \\ X_{n+1}^2 = X_n^2 + A_{n+1}^2. \end{cases}$$

The transition operator  $\hat{P}_{k,l}(i, j)$  is given by :

$$\hat{P}_{k,l}(i, j) = \begin{cases} \bullet \int_0^\infty \frac{(\lambda_2 t)^{j-l}}{(j-l)!} e^{-\lambda_2 t} b(t) dt, ; \text{if } i = k - 1, j \geq l, k \neq 0, l \geq 0, \\ \bullet \int_0^\infty \frac{(\lambda_2 t)^{j-l+1}}{(j-l+1)!} e^{-\lambda_2 t} b(t) dt, \text{if } 1 \leq l \leq j + 1, k = 0, i = 0, l \neq 0, \\ \bullet \int_0^\infty \frac{(\lambda_2 t)^j}{j!} e^{-\lambda_2 t} b(t) dt, \text{if } i \geq 0, j \geq 0, k = 0, l = 0, \\ \bullet 0 \text{ otherwise.} \end{cases}$$

To estimate the difference between the stationary distribution of the chain  $X_n = (X_{n+1}^1, X_{n+1}^2)$  in the  $M_2/G/1$  and  $\hat{X}_n = (\hat{X}_{n+1}^1, \hat{X}_{n+1}^2)$  in the  $M/G/1$  system, we apply the strong stability criterion.

### 3 Strong Stability in an $M_2/G/1$ Queue with Preemptive Priority

In this section, we determine the domain within the system  $M_2/G/1$  is strongly  $v$ -stable after a small perturbation of the intensity of the priority flux.

**Theorem 1.** *Let us denote  $\beta_0 = \sup(\beta : \hat{f}(\lambda\beta - \lambda) < \beta)$*

- $\lambda_2 E(U) < 1$
- $\exists a > 0$  such as  $E(e^{aU}) = \int_0^\infty e^{au} b(u) du < \infty$ .

Where introduce the following condition of ergodicity, then for all  $\beta$  such that  $1 < \beta \leq a$ , the imbedded Markov chain  $\hat{X}_n = (\hat{X}_n^1, \hat{X}_n^2)$  is strongly stable for the function  $v(i, j) = a^i \beta^j$ , where  $\alpha = \frac{\hat{f}(\lambda_2\beta - \lambda_2)}{\rho}$ ,  $\rho = \frac{\hat{f}(\lambda_2\beta - \lambda_2)}{\beta} < 1$  and  $\hat{f}(\lambda_2\beta - \lambda_2) = \int_0^\infty e^{(\lambda_2\beta - \lambda_2)u} b(u) du$ .

*Proof.* To be able to prove the  $v$ -stability of the  $M/G/1$  queue with priority we choose:

$$V(i, j) = a^i \times \beta^j, \alpha > 1, \beta > 1. \quad h(k, l) = 1_{(k=0, l=0)}$$

and  $\alpha(i, j) = \hat{P}_{0,0}(0, j)$ ; where  $\hat{P}_{0,0}(0, j) = \int_0^\infty \frac{(\lambda_2 t)^j}{j!} e^{-\lambda_2 t} b(t) dt$

We apply theorem □:

$$\hat{\pi}h = \sum_{k \geq 0} \sum_{l \geq 0} \hat{\pi}_{k,l}(i, j) h_{k,l} = \hat{\pi}_{0,0} = 1 - \frac{\lambda_2}{\mu} > 0.$$

$$\alpha 1 = \sum_{i \geq 0} \sum_{j \geq 0} \alpha(i, j) = \sum_{j \geq 0} \int_0^\infty \frac{(\lambda_2 t)^j}{j!} e^{-\lambda_2 t} b(t) dt = \int_0^\infty b(t) dt = 1.$$

$$\alpha h = \sum_{k \geq 0} \sum_{l \geq 0} \alpha_{k,l} h_{k,l} = \alpha_{0,0}(i, j) = \hat{P}_{0,0}(0, j) > 0.$$

**Verification of a.** We have two cases:  $\mathbf{k} = 0, \mathbf{l} = 0$

$$T_{0,0}(i, j) = \hat{P}_{0,0}(0, j) - 1\hat{P}_{0,0}(0, j) = 0 \geq 0.$$

Because ,

$$\sum_{i \geq 0} \sum_{j \geq 0} V(i, j) T_{k,l}(i, j) = \|TV(k, l)\|_v \leq \rho V(k, l).$$

$$\|h(k, l)\|_v = \sup_{k \geq 0} \sup_{l \geq 0} \frac{|h(k, l)|}{V(k, l)} = 1$$

$$\|\alpha(k, l)\|_v = \sum_{j \geq 0} a^0 \beta^j \hat{P}_{0,0}(0, j) = \sum_{j \geq 0} \int_0^\infty \frac{(\lambda_2 \beta t)^j}{j!} e^{-\lambda_2 t} b(t) dt = \hat{f}(\lambda \beta - \lambda)$$

Therefore,  $\|\hat{P}_{k,l}\| \leq 1 + \hat{f}(\lambda \beta - \lambda) < \infty$ .

### 3.1 Estimation of Stability

In order to obtain the error due to the approximation of the system  $M_2/G/1$  by the  $M/G/1$  one, let us estimate the norm of deviation of the transition kernel.

#### Estimation of Deviation of Transition Kernels

To estimate the margin between the stationary distribution of Markov chain  $\tilde{X}_n$  and  $X_n$ , first we estimate the norm of the deviation of transition kernels.

**Theorem 2.** For all  $\beta$  and  $\alpha$ , such as  $1 < \beta \leq \alpha$ ,  $\|\Delta\|_v = \|P - \hat{P}\|_v \leq D$ .  
Such that,

$$D = \max\{K_1, K_2, K_3\} \quad (2)$$

$$K_1 = \frac{\lambda_1}{\lambda_1 + \lambda_2} \int_0^\infty e^{(\alpha-1)\lambda_1 t} e^{(\beta-1)\lambda_2 t} b(t) dt + \frac{\lambda_2}{\lambda_1 + \lambda_2} \int_0^\infty e^{-\lambda_1 t} b(t) dt \int_0^\infty e^{(\beta-1)\lambda_2 t} b(t) dt$$

$$+ \frac{\lambda_2}{\lambda_1 + \lambda_2} \int_0^\infty \lambda_1 t e^{-\lambda_1 t} b^*(t) dt \int_0^\infty e^{(\beta-1)\lambda_2 t} b^*(t) dt$$

$$K_2 = \frac{1}{\beta} \int_0^\infty e^{-\lambda_1 t} b(t) dt \int_0^\infty e^{(\beta-1)\lambda_2 t} b(t) dt + \int_0^\infty \lambda_1 t e^{-\lambda_1 t} b^*(t) dt \int_0^\infty e^{(\beta-1)\lambda_2 t} b^*(t) dt$$

$$K_3 = \frac{1}{\alpha} \left[ \int_0^\infty e^{(\alpha-1)\lambda_1 t + (\beta-1)\lambda_2 t} b(t) dt + \int_0^\infty e^{\lambda_2(\beta-1)t} b(t) dt \right]$$

We must choose smallest of the estimates obtained in  $\{K_1, K_2, K_3\}$ .

These intermediate results allow us to consider the problem of obtaining estimates of stability, with an exact computation of the constants. for this, we introduce:

$\pi(i, j)$ : the joint stationary distribution of the process of number of request of the priority and non-priority of the system  $M_2/G/1$ .

$\hat{\pi}(i, j)$ : the joint stationary distribution of the process of the number of the non-priority request of the system  $M/G/1$

### Generating Function

In order to estimate the norm  $\|\hat{\pi}\|_v$ , necessary for the obtaining of the stability inequalities, let us calculate the generating function  $\Pi(Z_1, Z_2)$  of  $\hat{\pi}$ .

**Theorem 3.** *Let us note  $\Pi(Z_1, Z_2)$  the generating function of the  $\hat{\pi}(i, j)$  (stationary distribution of the  $M/G/1$  system). If the two conditions of ergodicity are verified:*

$$\begin{cases} \lambda_2 E(U) \leq 1 \\ \exists a > 0, \text{ such that } E(e^{au}) = \int_0^\infty e^{au} b(u) du < \infty. \end{cases}$$

We have the equality:  $\Pi(Z_1, Z_2) = \frac{(Z_2-1)\hat{f}(\lambda_2 Z_2 - \lambda_2)}{Z_2 - \hat{f}(\lambda_2 Z_2 - \lambda_2)} (1 - \frac{\lambda_2}{\mu})$ .

Where

$$\hat{f}(\lambda_2 Z_2 - \lambda_2) = \int_0^\infty e^{\lambda_2(Z_2-1)t} b(t) dt. \quad (3)$$

and

$$\mu = E(u) = \int_0^\infty t b(t) dt. \quad (4)$$

*Proof.*  $\Pi(Z_1, Z_2) = \sum_{i \geq 0} Z_1^i \Pi(i, Z_2) = \Pi(0, Z_2) + \sum_{i \geq 1} Z_1^i \Pi(i, Z_2)$

Where  $\Pi(i, Z_2) = \sum_{j \geq 0} \hat{\pi}(i, j) Z_2^j$ .

From the transition of the transition kernel, we have:

$$\begin{aligned} \hat{\pi}(i, j) &= \sum_k \sum_l \hat{\pi}(k, l) \hat{P}_{k,l}(i, j) = 1_{i=0} \hat{\pi}(0, j) \hat{P}_{0,0}(0, j) + 1_{i=0} \sum_{l>0} \hat{\pi}(0, l) \hat{P}_{0,l}(0, j) \\ &+ 1_{j \geq 0} \sum_{k>0} \left\{ \sum_{l=0}^j \hat{\pi}(k, l) \hat{P}_{k,l} \right\} 1_{i=k-1} \end{aligned}$$

**For  $i = 0$**

$$\begin{aligned} \hat{\pi}(0, j) &= \hat{\pi}(0, 0) \int_0^\infty \frac{(\lambda_2 t)^j}{j!} e^{-\lambda_2 t} b(t) dt + \sum_{l=1}^{j+1} \hat{\pi}(0, l) \int_0^\infty \frac{(\lambda_2 t)^{j-l+1}}{(j-l+1)!} e^{-\lambda_2 t} b(t) dt \\ &+ \sum_{l=0}^j \hat{\pi}(1, l) \int_0^\infty \frac{(\lambda_2 t)^{j-l}}{(j-l)!} e^{-\lambda_2 t} b(t) dt. \end{aligned}$$

Using the method of generating functions, we obtain:

$$\Pi(1, Z_2) = \Pi(0, Z_2) \frac{Z_2 - \hat{f}(\lambda_2 Z_2 - \lambda_2)}{Z_2 \hat{f}(\lambda_2 Z_2 - \lambda_2)} + \frac{1 - Z_2 \hat{f}(\lambda_2 Z_2 - \lambda_2)}{Z_2 \hat{f}(\lambda_2 Z_2 - \lambda_2)} \hat{\pi}(0, 0)$$

**For  $i > 0$**

$$\hat{\pi}(i, j) = \sum_{l \geq 0} \hat{\pi}(k, l) \int_0^\infty \frac{(\lambda_2 t)^{j-l}}{(j-l)!} e^{-\lambda_2 t} b(t) dt.$$

And,

$$\Pi(i, Z_2) = \Pi(i+1, Z_2) \hat{f}(\lambda_2 Z_2 - \lambda_2) \iff \Pi(i, Z_2) = \frac{\Pi(1, Z_2)}{\hat{f}^{i-1}(\lambda_2 Z_2 - \lambda_2)}$$

If there is no priority request in the system (when  $\theta = 0$ ), we are in case of the system  $M/G/1$ .

$$\Pi(0, Z_2) = \frac{(Z_2 - 1)\hat{f}(\lambda_2 Z_2 - 1)}{Z_2 - \hat{f}(\lambda_2 Z_2 - \lambda_2)} \hat{\pi}(0, 0) \quad (5)$$

This is the Pollatchek- Khinchin formula.

### 3.2 Inequality of Stability

In the following theorem, we calculate the error due to approximate  $M_2/G/1$  system by the  $M/G/1$  one on the stationary distribution.

**Theorem 4.** *Suppose that in a system  $M_2/G/1$  with preemptive priority, the conditions of theorem (1) hold. Then,  $\forall \beta$  and  $\alpha$ ,  $1 < \beta \leq \alpha$ ; we have the estimation*

$$\|\pi - \hat{\pi}\| \leq W_\theta, \quad (6)$$

Where,

$$W_\theta = D(1 + W)W(1 - \rho - (1 + W)D)^{-1},$$

$$W = (\beta - 1)(1 - \lambda_2/\mu) \frac{\rho}{1 - \rho},$$

$$D = \min\{K_1, K_2, K_3\}.$$

and  $\rho, \mu$  are respectively defined in (1), (4)

*Proof.* To verify the theorem (4), it is sufficient to estimate  $\|\pi\|_v$  and  $\|1\|_v$ , where 1 is the function identically equal to unity.  $\|\hat{\pi}\|_v = \sum_{i \geq 0} \sum_{j \geq 0} v(i, j) |\hat{\pi}(i, j)|$ , Where

$$v(i, j) = a^i \beta^j.$$

$$\text{From (5), we have, } \|\hat{\pi}\|_v = \frac{(\beta - 1)\hat{f}(\lambda_2 \beta - \lambda_2)}{\beta - \hat{f}(\lambda_2 \beta - \lambda_2)} (1 - \frac{\lambda_2}{\mu}) = W$$

$$\|1\|_v = \sup_{k \geq 0} \sup_{l \geq 0} \frac{1}{a^k \beta^l} \leq 1.$$

By definition,  $C = 1 + \|I\|_v \|\hat{\pi}\|_v = 1 + W$ . And,  $\|\Delta\|_v < \frac{1 - \rho}{C}$ .

Thence,

$$\|\pi - \hat{\pi}\|_v = D(1 + W)W(1 - \rho - (1 + W)D)^{-1}.$$

## 4 Conclusion

In this work, we are obtained the measurement and performance of the systems of queues with preemptive priority. We were interested in the study of strong stability in a system  $M_2/G/1$  with preemptive priority, after perturbation of the intensity of the arrivals of the priority requests. We clarified the conditions of approximation of the characteristics of the system of queue  $M_2/G/1$  with preemptive priority by those corresponding to the system of queue  $M/G/1$  classical. The method of strong stability also makes it possible to obtain the quantitative estimates of stability. We obtained the inequalities of stability with an exact calculation of the constants.

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# Stability of Two-Stage Queues with Blocking

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**Abstract.** Queueing networks are known to provide a useful modeling and evaluation tool in computer and telecommunications. Unfortunately, realistic features like finite capacities, retrials, priority, ... usually complicate or prohibit analytic solutions. Numerical and approximate computations as well as simplifications and performance bounds for queueing networks therefore become of practical interest. However, it is indispensable to delimit the stability domain wherever these approximations are justified.

In this paper we applied for the first time the strong stability method to analyze the stability of the tandem queues  $[M/G/1 \rightarrow ./M/1/1]$ . This enables us to determine the conditions for which the characteristics of the network with retrials  $[M/G/1/1 \rightarrow ./M/1/1]$ , can be approximated by the characteristics of the ordinary network  $[M/G/1 \rightarrow ./M/1/1]$  (without retrials).

**Keywords:** Queueing networks, tandem queues, Stability, Retrials, Blocking, Markov chain.

## 1 Introduction

Tandem queueing systems arise in mathematical modeling of computer and communication networks, manufacturing lines and other systems where customers, jobs, packets, ... , are subjected to a successive processing. Tandem queues can be used for modeling real-life two-node networks as well as for validation of general networks decomposition algorithms [6]. So, tandem queueing systems have found much interest in literature. The survey of early papers on tandem queues was done in [8]. The most of these papers are devoted to the exponential queueing models. Over the last two decades, efforts of many investigators in tandem queues were directed to study a complex two tandem queues. In particular, when priority, retrials, non-exponentiality of the service, ... arises in this networks. In this cases more often than not numerical and approximate computations as well as simplifications and performance bounds for queueing networks therefore become of practical interest. However, it is indispensable to delimit the stability domain wherever these approximations are justified.

The stability analysis of queueing networks have received a great deal of attention recently. This is partly due to several examples that demonstrate that the usual conditions "traffic intensities less than one at each station" are not sufficient

for stability, even under the well-known FIFO politics. Methods for establishing the stability of queueing networks have been developed by several authors, based on fluid limits [2], Lyapunov functions [5], explicit coupling (renovating event and Harris chains), monotonicity, martingales, large deviations, ....

The actual needs of practice require quantitative estimations in addition to the qualitative analysis, so in the beginning of the 1980's, a quantitative method for studying the stability of stochastic systems, called strong stability method "also called method of operators" was elaborated [1] (for full particulars on this method we suggest to see [9]). This method is applicable to all operation research models which can be governed by Markov chains.

In this article, we follow the strong stability approach to establish the stability of a two tandem queue with blocking in order to justify the approximation obtained by E. Moutzoukis and C. Langaris in [13].

The important feature and main originality of this work is that : since we establish the strong stability of the two tandem queues without intermediate space , the formulas for the characteristics of the ordinary model (without retrials) can be used to deduce the characteristics for the retrial model.

## 2 The Real Model

We consider a two single-server queues in tandem with blocking and retrials. Customers arrive at the first station, one a time, according to a poisson distribution with parameter  $\lambda$ . Each customer receives service at station 1 and then proceeds to station 2 for an additional service. There is no intermediate waiting room, so a customer whose service in the station 1 is completed can not proceed to the second station if the later is busy. Instead, the customer remains at station 1, so the last is blocked until station 2 becomes empty. The arriving customer who find the station 1 busy or blocked behave like retrial customer, he does not join a queue but he is placed instead in a hypothetical retrial queue of infinite capacity and retries for service under the constant retrial policy. According to this policy, the parameter of the exponential time of each customer in the retrial group is  $\frac{\mu}{n}$ , where  $n$  is the size of the retrial group. Thus, the total intensity is  $\mu$  (for the different interpretation of the constant retrial policy see Farahmand [4]). If the server of station 1 is free at the time of an attempt, then the customer at the head of the retrial group receives service immediately. Otherwise, he repeats his demand later.

The service times at stations 1 and 2 are independent and arbitrarily distributed random variables with probability density functions  $b_i(x)$ , distribution functions  $B_i(x)$  and finite mean values  $\mu_i$ , for  $i = 1, 2$ , respectively.

### 2.1 The General Process

Let  $X(t)$  represent the number of customers in the retrial box at time  $t$ , and for  $l = 1, 2$ :

$$\xi^l(t) = \begin{cases} 0 & \text{if the } l^{\text{th}} \text{ server is idle at time } t. \\ 1 & \text{if the } l^{\text{th}} \text{ server is working at time } t. \\ 2 & \text{if the } l^{\text{th}} \text{ server is blocked at time } t. \end{cases}$$

the considered model is completely describe by the regenerative process  $V(t) = (X(t), \xi^1(t), \xi^2(t))$ .

### 2.2 The Embedded Markov Chain $X_n$

Denote by  $d_n, n \in \mathbf{N}$ , the instant of the  $n^{\text{th}}$  departure from station 1. We assume, without loss of generality, that  $d_0 = 0$ . If we denote  $V_n = V(d_n + 0)$ , then it is clear that:  $V_n = (X(d_n + 0), \xi^1(d_n + 0), \xi^2(d_n + 0)) = (X_n, 0, 0)$ .

So, the process  $V_n$  is a semi regenerative process with embedded Markov renewal process  $(X, D) = \{X_n, d_n : n \in \mathbf{N}\}$ . The last process is an irreductible and aperiodic Markov chain with the probability matrix  $\mathbf{P} = \{p_{ij}\}$ , where:

$$p_{ij} = \begin{cases} \int_0^\infty \frac{(\lambda t)^j}{j!} e^{-\lambda t} f_0(t) dt, & \text{for } i = 0, \\ \int_0^\infty \frac{(\lambda t)^{j-i}}{(j-i)!} e^{-\lambda t} f_1(t) dt + \\ + \int_0^\infty \frac{(\lambda t)^{j-i+1}}{(j-i+1)!} e^{-\lambda t} f_2(t) dt, & \text{for } 1 \leq i < j + 1, \\ \int_0^\infty e^{-\lambda t} f_2(t) dt, & \text{for } i = j + 1, \\ 0, & \text{otherwise.} \end{cases}$$

where :

$$\begin{aligned} f_0(t) &= \int_0^\infty \lambda e^{-\lambda w} \frac{d}{dt} (B_1(t)B_2(t+w)) dw, \\ f_1(t) &= \int_0^\infty \lambda e^{-(\lambda+\mu)w} \frac{d}{dt} (B_1(t)B_2(t+w)) dw, \\ f_2(t) &= \int_0^\infty \mu e^{-(\lambda+\mu)w} \frac{d}{dt} (B_1(t)B_2(t+w)) dw. \end{aligned}$$

We define the function:  $\psi_u(s) = \int_0^\infty u e^{-uw} dw \int_0^\infty e^{-sx} dx (B_1(x)B_2(x+w))$  and we denote :

$$v_u = \left. \frac{-d\psi_u(s)}{ds} \right|_{s=0}; \rho^* = \frac{\lambda}{\lambda+\mu} + \lambda v_{\lambda+\mu}, \pi_k = \lim_{n \rightarrow \infty} P[X_n = k], k \in \mathbf{N},$$

If the intensity of the system  $\rho^* < 1$ , the Markov chain  $X_n$  is positive recurrent. In this case, the generating function  $\Pi(z) = \sum_{n=0}^\infty \pi_n z^n$  is given by:

$$\Pi(z) = \frac{z\psi_\lambda(\lambda - \lambda z) - \left(\frac{\lambda z + \mu}{\lambda + \mu}\right) \psi_{\lambda+\mu}(\lambda - \lambda z)}{z - \left(\frac{\lambda z + \mu}{\lambda + \mu}\right) \psi_{\lambda+\mu}(\lambda - \lambda z)} \pi_0, \tag{1}$$

$$\text{where: } \pi_0 = \lim_{n \rightarrow \infty} P[X_n = 0] = \frac{1 - \rho^*}{1 - \rho^* + \lambda v_\lambda}. \tag{2}$$

### 3 The Ideal Model

We assume that the mean retrial rate in our real model tends to infinity. So, the customers in the retrial orbite try continuously to find a position for service

and they become ordinary customers. It means that if  $\mu \rightarrow \infty$ , our real model becomes the simple model of two queues in tandem without intermediate room, it will be referred to as an ideal model.

Now, let  $\bar{X}(t)$  denote the number of customers in the first queue of the ideal model at time  $t$  and for  $l = 1, 2$  we consider:

$$\bar{\xi}^l(t) \begin{cases} 0 & \text{if the } l^{th} \text{ server is idle at time } t. \\ 1 & \text{if the } l^{th} \text{ server is working at time } t. \\ 2 & \text{if the } l^{th} \text{ server is blocked at time } t. \end{cases}$$

Our ideal model is completely described by:  $\bar{V}(t) = (\bar{X}(t), \bar{\xi}^1(t), \bar{\xi}^2(t))$ .

### 3.1 The Embedded Markov Chain $\bar{X}_n$

It is clear that  $\bar{X}_n = (\bar{X}, D) = \{\bar{X}_n, \bar{d}_n, n \geq 0\}$  is the embedded Markov renewal process of the semiregenerative process  $(\bar{X}(t), \bar{\xi}^1(t), \bar{\xi}^2(t))$ . We suppose that the intensity of the system  $\rho < 1$ , then  $\bar{X}_n$  is an irreducible and aperiodic recurrent Markov chain with transition probability matrix  $\bar{P} = \{\bar{p}_{ij}\}$ , where:

$$\bar{p}_{ij} = \begin{cases} \int_0^\infty \frac{(\lambda t)^j}{j!} e^{-\lambda t} f_0(t) dt, & i = 0, \\ \int_0^\infty \frac{(\lambda t)^{j-k+1}}{(j-k+1)!} e^{-\lambda t} d_t (B_1(t)B_2(t)), & 1 \leq i \leq j + 1, \\ 0, & \text{otherwise.} \end{cases}$$

In this case the generating function of the v.a.  $\bar{X}$  is defined as:

$$\bar{\Pi}(z) = \lim_{\mu \rightarrow \infty} \Pi(z) = \frac{z\psi_\lambda(\lambda - \lambda z) - \psi(\lambda - \lambda z)}{z - \psi(\lambda - \lambda z)} \bar{\pi}_0.$$

$$\bar{\pi}_0 = \frac{1 - \rho}{1 - \rho + \lambda v_\lambda}, \quad \psi(s) = \int_0^\infty e^{-st} d_t (B_1(t)B_2(t)),$$

$\rho = -\lambda \frac{d\psi(s)}{ds} \Big|_{s=0} = \lambda \int_0^\infty t d_t (B_1(t)B_2(t))$ , We suppose that the retrial rate tends to infinity and to characterize the proximity of the ideal and real model we define the variation distance:  $W = \int_0^{+\infty} |f_2(t) - \frac{d}{dt} (B_1(t)B_2(t))| dt$ .

## 4 The Strong Stability

This section contains preliminary results that are needed in the constructive proofs of the main theorems, given in the next sections. Let  $(E, \varepsilon)$ , a measurable space, where  $\varepsilon$  is a  $\sigma$ -algebra denumerably engendered. We consider a homogeneous Markov chain  $Y = (Y_t, t \geq 0)$  in the space  $(E, \varepsilon)$ , given by a transition kernel  $\mathcal{P}(\xi, \mathcal{A}), \xi \in \mathcal{E}, \mathcal{A} \in \varepsilon$  and having a unique invariant probability  $\nu$ .

Denote by  $m\varepsilon(m\varepsilon^+)$  the space of finite (nonnegative) measures on  $\varepsilon$  and by  $f\varepsilon(f\varepsilon^+)$  the space of bounded measurable (nonnegative) functions on  $E$ . We

associate to every transition kernel  $P(x, A)$  in the space of bounded operators, the linear mappings  $\mathcal{L}_P$  and  $\mathcal{L}_P^*$  defined by :

$$\begin{aligned} \mathcal{L}_P : \varepsilon &\rightarrow m\varepsilon & \mathcal{L}_P^* : f\varepsilon &\rightarrow f\varepsilon \\ \mu &\rightarrow \int_E \mu(dx)P(x, A), A \in \varepsilon & f &\rightarrow \int_E P(x, dy)f(y), x \in E. \end{aligned}$$

We also associate to every function  $f \in f\varepsilon$  the linear functional  $f : \mu \rightarrow \mu f$  such that:  $\mu f = \int_E \mu(dx)f(A); x \in E, A \in \varepsilon.$

We denote by  $f \circ \mu$  the transition kernel defined as the tensorial product of the measure  $\mu$  and the measurable function  $f$  having the form:

$$f(x)\mu(A); x \in E, A \in \varepsilon.$$

We consider, the Banach space  $M = \{\mu \in m\varepsilon / \|\mu\| < \infty\}$ , in the space  $m\varepsilon$  defined by a norm  $\|\cdot\|$  compatible with the structural order in  $m\varepsilon$ , i.e. :

$$\begin{aligned} \|\mu_1\| &\leq \|\mu_1 + \mu_2\|, \text{ for } \mu_i \in M^+, i = 1, 2, \\ \|\mu_1\| &\leq \|\mu_1 - \mu_2\|, \text{ for } \mu_i \in M^+, i = 1, 2; \mu_1 \perp \mu_2, \\ |\mu|(E) &\leq k\|\mu\|, \text{ for } \mu \in M, \end{aligned}$$

where  $|\mu|$  is the variation of the measure  $\mu$ ,  $k$  is a finite constant and  $M^+ = m\varepsilon^+ \cap M.$

The family of norms  $\|\mu\|_v = \int_E v(x) |\mu|(dx), \forall \mu \in m\varepsilon$ , where,  $v$  is a measurable function (not necessary finite) bounded from bellow by a positive constant, satisfy the above conditions. With this family of norms we can induce on the spaces  $f\varepsilon, M$  the following norms:

$$\|P\|_v = \sup\{\|\mu P\|_v, \|\mu\|_v \leq 1\} = \sup_{x \in E} \frac{1}{v(x)} \int_E |P(x, dy)| v(y), \tag{3}$$

$$\|f\|_v = \sup\{|\mu f|, \|\mu\|_v \leq 1\} = \sup_{x \in E} \frac{1}{v(x)} |f(x)|. \tag{4}$$

**Definition 1.** [1] We say that the Markov chain  $Y$ , with a bounded transition kernel  $\mathcal{P}$ , and a unique stationary measure  $\nu$ , is strongly  $v$ -stable if every stochastic kernel  $Q$  in the neighborhood  $\{Q : \|Q - \mathcal{P}\|_v \leq \epsilon\}$  admits a unique stationary measure  $\bar{\nu}$  and :

$$\|\nu - \bar{\nu}\|_v \rightarrow 0 \text{ when } \|Q - \mathcal{P}\|_v \rightarrow 0$$

**Theorem 1.** [1] The Harris recurrent Markov chain  $Y$  with a bounded transition kernel  $\mathcal{P}$ , and a unique stationary measure  $\nu$ , is strongly  $v$ -stable, if the following conditions holds:

1.  $\exists \alpha \in M^+, \exists h \in f\varepsilon^+ / \pi h > 0, \alpha \mathbf{I} = 1, \alpha h > 0,$
2.  $T = \mathcal{P} - h \circ \alpha \geq 0,$
3.  $\exists \gamma < 1 / Tv(x) \leq \gamma v(x), \forall x \in E,$

where  $\mathbf{I}$  is the function identically equal to 1.

**Theorem 2.** Under the conditions of the theorem (1) and if  $\Delta$  (the deviation of the operator transition  $\mathcal{P}$ ) verifying the condition  $\|\Delta\|_v < \frac{1-\gamma}{C}$ , we have:  
 $\|\nu - \bar{\nu}\|_v \leq \|\Delta\|_v \|\nu\|_v C(1 - \gamma - C\|\Delta\|_v)^{-1}$ ,  $C = 1 + \|\mathbf{I}\|_v \|\nu\|_v$ .

### 5 Stability of the Ideal Model

We define on  $E = \mathbf{N}$  the  $\sigma$ -algebra  $\varepsilon$  engendered by the set of all singletons  $\{j\}, j \in \mathbf{N}$ . We consider the function  $v(k) = \beta^k, \beta > 1$  and we define the norm:  $\|\mu\|_v = \sum_{j \in \mathbf{N}} v(j) |\mu|(\{j\}), \forall \mu \in m\varepsilon$ . We also consider the measure  $\alpha(\{j\}) = \alpha_j = \bar{p}_{0j}$ , and the measurable function  $h(i) = \begin{cases} 1 & \text{if } i = 0, \\ 0 & \text{otherwise.} \end{cases}$

Using the assumptions we obtain the following lemmas

**Lemma 1.** Let  $\bar{\pi}$  the stationary distribution of the Markov chain  $\bar{X}_n$ , then:

$$\alpha \mathbf{I} = 1, \quad \alpha h > 0 \text{ and } \alpha \bar{\pi} > 0.$$

*Proof.* It is easy to show that :

- $\alpha \mathbf{I} = \sum_{j=0}^{\infty} \alpha(\{j\}) = \sum_0^{\infty} p_{0j} = 1.$
- $\alpha h = \sum_{j=0}^{\infty} \alpha(\{j\})h(j) = p_{00} > 0.$
- $\bar{\pi}h = \sum_{i=0}^{\infty} \bar{\pi}_i h(i) = \bar{\pi}_0 = \frac{1-\rho}{1-\rho+\lambda v_\lambda} > 0.$

**Lemma 2.** Suppose that the following conditions holds:

1.  $\lambda \int_0^{+\infty} u d(B_1(u)B_2(u)) < 1,$  (Geometric ergodicity condition).
2.  $\exists a > 0 / \int_0^{\infty} e^{au} d(B_1(u)B_2(u)) < +\infty,$  (Cramer condition).
3.  $\int_0^{+\infty} t |f_2(t) - \frac{d}{dt}(B_1(t)B_2(t))| dt < \frac{W}{\lambda}.$

then,  $\exists \beta > 1$  such that:

$$- \frac{\psi(\lambda - \lambda\beta)}{\beta} < 1. \quad \bullet \int_0^{+\infty} e^{(\lambda\beta - \lambda)t} |f_2(t) - \frac{d}{dt}(B_1(t)B_2(t))| dt < \beta W.$$

Where  $W$  is given by the formula (3.1).

*Proof.*

• We consider the function:  $K(\beta) = \psi(\lambda - \lambda\beta)$ .  $K$  is continuous differentiable in  $[1, a]$ , so:  $K'(\beta) = \lambda \int_0^{+\infty} t e^{(\lambda\beta - \lambda)t} d_t(B_1(t)B_2(t))$ ,

$$K''(\beta) = \lambda^2 \int_0^{+\infty} t^2 e^{(\lambda\beta - \lambda)t} d_t(B_1(t)B_2(t)),$$

then,  $K$  is a strictly convex function in  $[1, a]$ . We define the function :

$$L(\beta) = \frac{K(\beta)}{\beta} = \frac{1}{\beta} \int_0^{+\infty} e^{(\lambda\beta - \lambda)t} d_t(B_1(t)B_2(t)).$$

For  $\beta = 1, L(1) = \psi(0) = 1$  and for  $1 < \beta < a$ ,  $L'(\beta) = \frac{\lambda\beta\psi(\lambda - \lambda\beta) - \psi(\lambda - \lambda\beta)}{\beta^2}$ ,

From the first assumption we have:

$$L'(1) = \lambda\psi'(0) - \psi(0) = \lambda \int_0^{+\infty} u d(B_1(u)B_2(u)) - \psi(0) < 0.$$

So in the vicinity of 1,  $L$  is decreasing. Then,  $\exists \beta > 1$  such that:

$L(\beta) < L(1) \Rightarrow L(\beta) < 1$ , so :  $\exists \beta > 1 : \frac{\psi(\lambda - \lambda\beta)}{\beta} < 1$ . Moreover, let's consider:

$$\beta_0 = \sup \{ \beta : \psi(\lambda\beta - \lambda) < 1 \}, 1 < \beta_0 < +\infty. \tag{5}$$

The convexity of the function  $K(\beta)$  imply that :

$$\psi(\lambda\beta - \lambda) < \beta, \forall \beta \in ]1, \beta_0 ] \Rightarrow \psi(\lambda\beta - \lambda) < \beta_0,$$

• We put :  $\varphi(\lambda\beta - \lambda) = \int_0^{+\infty} e^{(\lambda\beta - \lambda)t} | f_2(t) - \frac{d}{dt} (B_1(t)B_2(t)) | dt$  and we consider the function  $\Omega(\beta) = \varphi(\lambda - \lambda\beta)$ .

For  $\beta = 1$  :  $\Omega(1) = \varphi(0) = \int_0^{+\infty} | f_2(t) - \frac{d}{dt} (B_1(t)B_2(t)) | dt = W$ .

For  $1 < \beta < a$ ,  $\Omega$  is continuous and differentiable, so:  $\Omega'(\beta) = \frac{\beta\varphi'(\lambda\beta - \lambda) - \varphi(\lambda\beta - \lambda)}{\beta^2}$ .  
The functions  $\varphi$  and  $\varphi'$  are continuous, then:

$$\lim_{\beta \rightarrow 1^+} \Omega'(\beta) = \lim_{\beta \rightarrow 1^+} [\lambda\varphi'(\lambda\beta - \lambda) - \varphi(\lambda - \lambda\beta)] = \lambda\varphi'(0^+) - \varphi(0^+) \tag{6}$$

$\varphi'(0^+) = \int_0^{+\infty} t | f_2(t) - \frac{d}{dt} (B_1(t)B_2(t)) | dt$  and  $\varphi(0^+) = W$ . From the third assumption we have :

$$\lambda \int_0^{+\infty} t | f_2(t) - \frac{d}{dt} (B_1(t)B_2(t)) | dt < W,$$

so  $\Omega'(1^+) < 0$  then  $\varphi(\beta) - \varphi(1) < 0$  in the vicinity of 1. It means that  $\exists \beta > 1$  such that  $\Omega(\beta) = \frac{\varphi(\lambda\beta - \lambda)}{\beta} < W$ .

**Lemma 3.** *The operator  $\bar{T} = \bar{P} - h \circ \alpha$  is nonnegative and  $\exists \gamma < 1$ , such that  $\bar{T}v(k) \leq \gamma v(k)$  for all  $k \in \mathbf{N}$ .*

*Proof.* We have  $\bar{T}(i, \{j\}) = \bar{T}_{ij} = \bar{p}_{ij} - h(i)\alpha(\{j\})$ , so :

$$\bar{T}_{ij} = \begin{cases} 0, & \text{if } i = 0, \\ \bar{p}_{ij} \geq 0, & \text{if } i \geq 1. \end{cases} \Rightarrow \bar{T} \text{ is non negative.}$$

Let's compute  $\bar{T}v(k)$ :

If  $k = 0$ , we have  $\bar{T}v(0) = 0$ . If  $k \neq 0$ , we have :

$$\begin{aligned} \bar{T}v(k) &= \sum_{j \geq 0} \beta^j T_{kj}, 1 \leq k \leq j + 1, \\ &= \beta^{k-1} \int_0^{+\infty} e^{(\lambda\beta - \lambda)t} dt (B_1(t)B_2(t)) = \beta^{k-1} \psi(\lambda - \lambda\beta). \end{aligned}$$

We consider that  $\gamma = \frac{\psi(\lambda - \lambda\beta)}{\beta}$ . From the lemma (2),  $\exists \beta \in ]1, \beta_0]$  such that  $\gamma < 1$ . So, there exists  $\beta$  with  $1 < \beta \leq \beta_0$  such that:

$$\bar{T}v(k) \leq \gamma v(k), \forall k \in \mathbf{N}, \gamma = \frac{\psi(\lambda - \lambda\beta)}{\beta} < 1. \tag{7}$$

**Lemma 4.** *The norm of the transition kernel of the chain  $\bar{X}_n$  is bounded.*

*Proof.* We have:  $\|\bar{P}\| = \|\bar{T} + h \circ \alpha\|_v \leq \|\bar{T}\|_v + \|h\|_v \|\alpha\|_v$ .

$$\bullet \|\bar{T}\|_v = \sup_{k \geq 0} \frac{1}{v(k)} \sum_{j \geq 0} v(j) | \bar{T}_{kj} | = \gamma < 1,$$

$$\bullet \|h\|_v = \sup_{k \geq 0} 1/v(k) = \frac{1}{\beta^k} = 1,$$

$$\bullet \|\alpha\|_v = \sum_{j \geq 0} \beta^j \int_0^\infty \frac{(\lambda t)^j}{j!} e^{-\lambda t} f_0(t) dt = \int_0^\infty e^{(\beta\lambda - \lambda)t} dF(t) < \infty,$$

with  $F(t) = \int_0^\infty \lambda e^{-\lambda w} (B_1(t)B_2(t+w)) dw.$

So:  $\|\bar{P}\|_v \leq 1 + \beta_0 < \infty.$

**Theorem 3.** *In the two tandem queues with blocking, the Markov chain  $\bar{X}_n$  representing the number of customers in the first station at the instant of the  $n^{th}$  departure from the first station, is strongly  $v$ -stable with respect to the function  $v(k) = \beta^k$  for all  $1 < \beta \leq \beta_0$ . Where  $\beta_0$  is given by the formula (5).*

*Proof.* The proof arises from the theorem 1. Indeed, all necessary conditions to establish the  $v$ -strong stability required in the theorem 1 are satisfied and are given by the above lemmas (1), (3), (4).

### 6 Deviation of the Transition Operator

**Lemma 5.** *Let  $P$  (resp.  $\bar{P}$ ) be the transition operator associate to the Markov chain  $X_n$  ( resp.  $\bar{X}_n$ ). Then:  $\|P - \bar{P}\|_v \leq W + \int_0^\infty e^{(\beta\lambda - \lambda)t} f_1(t) dt.$*

*Proof.* We have :

$$\begin{aligned} \|P - \bar{P}\|_v &= \sup_{k \geq 0} \frac{1}{v(k)} \sum_{j \geq 0} v(j) |p_{kj} - \bar{p}_{kj}|, \sup_{k \geq 0} \frac{1}{\beta^k} \sum_{j \geq 0} \beta^j |p_{kj} - \bar{p}_{kj}|, \\ &= \sup \left( 0, \sup_{k > 0} \frac{1}{\beta^k} \sum_{j \geq 0} \beta^j |p_{kj} - \bar{p}_{kj}| \right), \end{aligned}$$

We put  $Q(k) = \sum_{j \geq 0} \beta^j |p_{kj} - \bar{p}_{kj}|$ . If  $k \neq 0$  we have  $1 \leq k \leq j + 1$ , so :

$$\begin{aligned} Q(k) &= \sum_{j \geq 0} \beta^j |p_{kj} - \bar{p}_{kj}| = \sum_{j \geq k-1} \beta^j |p_{kj} - \bar{p}_{kj}|, \\ &= \beta^{k-1} |p_{kk-1} - \bar{p}_{kk-1}| + \sum_{j \geq k} \beta^j |p_{kj} - \bar{p}_{kj}|, \\ &\leq \beta^k \left[ \frac{\int_0^\infty e^{(\beta\lambda - \lambda)t} |f_2(t) - \frac{d}{dt} (B_1(t)B_2(t))| dt}{\beta} + \int_0^\infty e^{(\beta\lambda - \lambda)t} f_1(t) dt \right]. \end{aligned}$$

Using the lemma (2), we obtain:

$$\|P - \bar{P}\|_v \leq W + \int_0^\infty e^{(\beta\lambda - \lambda)t} f_1(t) dt, \text{ with: } \lim_{\mu \rightarrow \infty} \int_0^\infty e^{(\beta\lambda - \lambda)t} f_1(t) dt = 0.$$

### 7 Deviation of the Stationary Distribution

**Theorem 4.** *Let's  $\pi$  (resp.  $\bar{\pi}$ ) the stationary distribution of the real model,  $[M/G/1/1 \rightarrow ./G/1/1]$  with retrials, (resp. the ideal model  $[M/G/1 \rightarrow ./G/1/1]$ ).*



For  $1 < \beta < \beta_0$  and  $\|\Delta\|_v < \frac{1-\gamma}{1+c_0}$ , we have:

$$\|\pi - \bar{\pi}\|_v \leq c_0(1+c_0)\|\Delta\|_v(1-\gamma - (1+c_0)\|\Delta\|_v)^{-1}, \text{ where } c_0 = \frac{\psi_\lambda(\lambda\beta-\lambda)-\gamma}{1-\gamma}.$$

*Proof.* From the theorem (2) we have :

$$\|\pi - \bar{\pi}\|_v \leq \|\Delta\|_v \|\bar{\pi}\|_v C(1-\gamma - C\|\Delta\|_v)^{-1}.$$

Or we have :  $\|\bar{\pi}\|_v = \sum_{j \geq 0} v(j)\bar{\pi} = \bar{\Pi}(\beta)$   
 $= \frac{\beta\psi_\lambda(\lambda\beta-\lambda) - \psi(\lambda\beta-\beta)}{\beta - \psi(\lambda\beta-\lambda)} = \frac{\psi_\lambda(\lambda\beta-\lambda) - \gamma}{1-\gamma} = c_0.$

and  $\|\mathbf{I}\|_v = \sup_{k \geq 0} \frac{1}{\beta^k} = 1$  So:

$$C = 1 + \|\mathbf{I}\|_v \|\bar{\pi}\|_v = 1 + \frac{\psi_\lambda(\lambda-\lambda\beta) - \gamma}{1-\gamma} = 1 + c_0. \text{ Finally, we obtain :}$$

$$\|\pi - \bar{\pi}\|_v \leq c_0(1+c_0)\|\Delta\|_v(1-\gamma - (1+c_0)\|\Delta\|_v)^{-1}.$$

## 8 Conclusion

This work is a first attempt to prove the applicability of the strong stability method to a queueing networks. We have obtained the conditions under which the characteristics of the tandem queues  $[M/G/1/1 \rightarrow ./M/1/1]$  with retrials can be approximated by those of the ordinary network  $[M/G/1 \rightarrow ./M/1/1]$  (without retrials). This allow us to justify the approximation established by E. Moutzoukis and C. Langaris in [13].

In term of prospect, we propose to work out an algorithm which checks the conditions of approximation of these two tandem queues and determine with precision the values for which the approximation is possible. It will also determine the error on the stationary distribution which had with the approximation.

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# Performance Evaluation in a Queueing System $M_2/G/1$

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**Abstract.** In this communication, we use the strong stability method to approximate the characteristics of the  $M_2/G/1$  queue with preemptive resume priority by those of the  $M/G/1$  one. For this, we first prove the stability fact and next obtain quantitative stability estimates with an exact computation of constants.

**Keywords:** Strong stability, Approximation, Preemptive priority, Markov chain.

## Stability of Two-Stage Queues with Blocking

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**Abstract.** Queueing networks are known to provide a useful modeling and evaluation tool in computer and telecommunications. Unfortunately, realistic features like finite capacities, retrials, priority, ... usually complicate or prohibit analytic solutions. Numerical and approximate computations as well as simplifications and performance bounds for queueing networks therefore become of practical interest. However, it is indispensable to delimit the stability domain wherever these approximations are justified.

In this paper we applied for the first time the strong stability method to analyze the stability of the tandem queues  $[M/G/1 \rightarrow ./M/1/1]$ . This enables us to determine the conditions for which the characteristics of the network with retrials  $[M/G/1/1 \rightarrow ./M/1/1]$ , can be approximated by the characteristics of the ordinary network  $[M/G/1 \rightarrow ./M/1/1]$  (without retrials).

**Keywords:** Queueing networks, tandem queues, Stability, Retrials, Blocking, Markov chain.