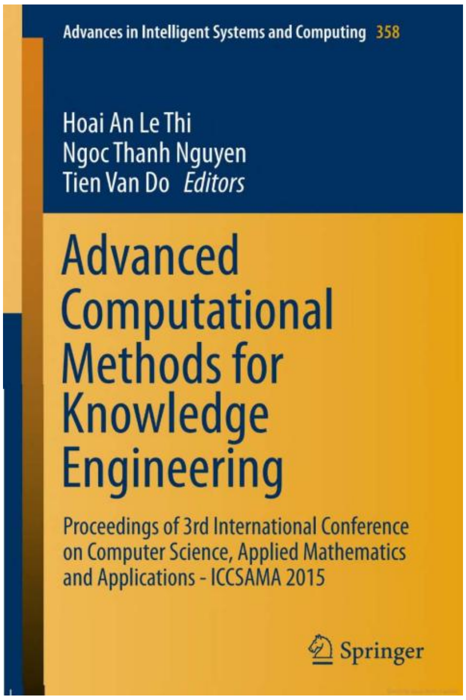

Quality of the Approximation of Ruin Probabilities Regarding to Large Claims

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1 Introduction

In the actuarial literature, the evolution in time of the capital of an insurance company is often modeled by the process of reserve resulting from the difference between the premium-income and the pay-out process.

The probability of ruin is one of the basic characteristics of risk models and various authors investigate the problem of its evaluation (for example, see [1] and [11], Chapter 11). However, it cannot be found in an explicit form for many risk models. Furthermore, parameters governing these models are often unknown and one can only give some bounds for their values. In such a situation the question of stability becomes crucial.

Indeed, when using a stochastic model in insurance mathematics one has to consider this model as an approximation of the real insurance activities. The stochastic elements derived from these models represent an idealization of the real insurance phenomena under consideration. Hence the problem arises out of establishing the limits in which we can use our 'ideal' model. The practitioner has to know the accuracy of his recommendations, resulting from his investigations based on the ideal model [2]. Using approximations means here that we investigate 'ideal' models which are rather simple, but nevertheless close in some sense to the real (disturbed) model.

After introducing the problem of stability in insurance mathematics by Beirlant and Rachev [2], Kalashnikov [7] investigated the estimation of ruin probabilities in the univariate risk models, using the strong stability method, the reversed process notion and the supplementary variables technique.

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On the other hand, we often deal in insurance and finance with large claims that are described by heavy-tailed distributions (Pareto, Lognormal, Weibull, ...). It is worthy of notice the special importance of heavy-tailed distributions, which is increasing the last years because of occasional appearance of huge claims [4,9,5,6,12]. Indeed, the loss distribution in actuarial science and financial risk management is fundamental and of ultimate use. It describes the probability distribution of payment to the insured. In most situations losses are small, and extreme losses rarely occur. But the number and the size of the extreme losses can have a substantial influence on the profit of the company. Traditional methods in actuarial literature use parametric specifications to model loss distributions by a single parametric model or decide to analyze large and small losses separately. The most popular specifications are the lognormal, Weibull and Pareto distributions or a mixture of lognormal and Pareto distributions.

The aim of this work is to study, on the basis of numerical examples based on simulation results, the sensitivity of the strong stability bound on ruin probabilities established by Kalashnikov [7] regarding to the different heavy-tailed distributions mentioned above.

2 Strong Stability of a Univariate Classical Risk Model

2.1 Description of the Model

The classical risk process in the one-dimensional situation can be stated as

$$X(t) = u + ct - Z(t), \quad t \geq 0, \tag{1}$$

where $X(t)$ is the surplus of an insurance company at time $t \geq 0$, $u \geq 0$ the initial surplus, c the rate at which the premiums are received, and $Z(t)$ the aggregate of the claims between time 0 and t. $Z(t) = \sum_{i=1}^{N(t)} Z_i$, where $\{Z_i, i \geq 1\}$ is a sequence of iid random variables, representing the claim amounts of distribution function denoted by $F(x)$ and mean claim size denoted by μ, $\{N(t), t \geq 0\}$ being a Poisson process with parameter λ, representing the number of claims. The relative security loading θ is defined by $\theta = \frac{c-\lambda\mu}{\lambda\mu}$. We further assume that $c > \lambda\mu$, the expected payment per unit of time.

Ruin theory for the univariate risk process defined as (1) has been extensively discussed in the literature (for example, see [1] and [11], Chapter 11).

Let us denote the reversed process associated to the risk model by $\{V_n\}_{n\geq 0}$. The strong stability approach consists of identifying the ruin probability $\Psi_a(u)$ associated to the risk model governed by a vector parameter $a = (\lambda, \mu, c)$, with the stationary distribution of the reversed process $\{V_n\}_{n\geq 0}$ [7], i.e.

$$\Psi_a(u) = \lim_{n\to\infty} \mathbb{P}(V_n > u),$$

where u is the initial reserve.

2.2 Strong Stability of a Univariate Classical Risk Model

For a general framework on the strong stability method, the reader is referred to [8]. However, let us recall the following basic definition.

Definition 1. *[8] A Markov chain X with transition kernel P and invariant measure π is said to be υ-strongly stable with respect to the norm $\|.\|_\upsilon$ ($\|\alpha\|_\upsilon = \int_0^\infty \upsilon(x)|\alpha|(dx)$, for a measure α), if $\|P\|_\upsilon < \infty$ and each stochastic kernel Q in some neighborhood $\{Q : \|Q - P\|_\upsilon < \epsilon\}$ has a unique invariant measure $\mu = \mu(Q)$ and $\|\pi - \mu\|_\upsilon \to 0$ as $\|Q - P\|_\upsilon \to 0$.*

More concrete, following the preceding definition, our approximation problem can be stated in the following way: if the input elements of the ideal and real models are 'close' to each other, then, can we estimate the deviation between the corresponding outputs? In other words, the stability theory in general renders the following picture: If we have as input characteristics the distribution function of the service times (claims distribution function for our risk model) and as output characteristics the stationary distribution of the waiting times (ruin probability for our risk model), the stability means that the convergence in $\mathcal{L}^1$ of the input characteristics implies the weak convergence of the output characteristics.

Let $a' = (\lambda', \mu', c')$ be the vector parameter governing another univariate risk model defined as above, its ruin probability and its reversed process being respectively $\Psi_{a'}(u)$ and $\{V'_n\}_{n\geq 0}$.

The following theorem determines the υ-strong stability conditions of a univariate classical risk model. It also gives the estimates of the deviations between both transition operators and both ruin probabilities in the steady state.

Theorem 1. *[7] Consider a univariate classical risk model governed by a vector parameter a. Then, there exists $\varepsilon > 0$ such that the reversed process $\{V_n\}_{n\geq 0}$ (Markov chain) associated to this model is strongly stable with respect to the weight function $\upsilon(x) = e^{\epsilon x}$ $(\epsilon > 0)$, $x \in \mathbb{R}^+$.*

In addition, if $\mu(a, a') < (1 - \rho(\epsilon))^2$, then we obtain the margin between the transition operators P and P' of the Markov chains $\{V_n\}_{n\geq 0}$ and $\{V'_n\}_{n\geq 0}$:

$$\|P - P'\|_\upsilon \leq 2\mathbb{E}e^{\epsilon Z}|ln\frac{\lambda c'}{\lambda' c}| + \|F - F'\|_\upsilon,$$

where,

$$\mu(a, a') = 2\mathbb{E}e^{\epsilon Z}|ln\frac{\lambda c'}{\lambda' c}| + \|F - F'\|_\upsilon,$$
$$\rho(\epsilon) = \mathbb{E}(\exp\{\epsilon(Z_1 - c\theta_1)\}),$$
$$\|F - F'\|_\upsilon = \int_0^\infty \upsilon(u)|d(F - F')|(u) = \int_0^\infty \upsilon(u)|f - f'|(u)du.$$

Moreover, we have the deviation between the ruin probabilities:

$$\|\Psi_a - \Psi_{a'}\|_\upsilon \leq \frac{\mu(a, a')}{(1 - \rho(\epsilon))((1 - \rho(\epsilon))^2 - \mu(a, a'))} = \Gamma. \tag{2}$$

Remark 1. Without loss of generality, we relax some conditions by taking $\lambda' = \lambda$ and $c' = c$, then we have: $\mu(a, a') = \|F - F'\|_v = \int_0^\infty v(u)|f - f'|(u)du$. The perturbation may concern the mean claim size parameter (i.e. $\mu' = \mu + \varepsilon$) or the claim amounts distribution function F itself.

3 Simulation Based Study

We want to analyze the quality and the sensitivity of the bound defined as in formula (2) of Theorem 1 regarding to certain heavy-tailed distributions. To do so, we elaborated an algorithm which follows the following steps:

3.1 Algorithm

1) Introduce the parameters λ, μ, c of the ideal model, and λ', μ', c' of the perturbed (real) model.
2) Verify the positivity of the relative security loadings θ and θ' defined by: $\theta = \frac{c - \lambda\mu}{\lambda\mu}$ and $\theta' = \frac{c' - \lambda'\mu'}{\lambda'\mu'}$.
If yes, (*the ruin of the models is not sure*) go to step 3;
else return to step 1.
3) Initialize ϵ ($\epsilon > 0$) such that $0 < \rho(\epsilon) < 1$ and Γ be minimal.
4) Compute $\mu(a, a') = \int_0^\infty v(u)|f - f'|(u)du$, and test:
$\mu(a, a') < (1 - \rho(\epsilon))^2$.
If yes, (*we can deduce the strong stability inequality*) go to step 5;
else increment ϵ with step p, then return to step 4.
5) Compute the bound Γ on the deviation $\|\Psi_a - \Psi_{a'}\|_v$ such that:

$$\|\Psi_a - \Psi_{a'}\|_v \leq \frac{\mu(a, a')}{(1 - \rho(\epsilon))((1 - \rho(\epsilon))^2 - \mu(a, a'))} = \Gamma.$$

Using the above algorithm, we perform a comparative study (comparison of the resulting error on ruin probabilities) based on simulation results obtained with the following different distributions.

3.2 Simulated Distributions

In this section, we compare the following four distributions (Lognormal, Weibull, logistic, mixture (Lognormal-Pareto)). In order to well discuss and judge our results, we also use a benchmark distribution the exponential one (see Table 1).

1. The density of the Lognormal law

$$f(t/\alpha, \beta) = \frac{1}{t\beta\sqrt{2\pi}} e^{-\frac{(\log(t) - \alpha)^2}{2\beta^2}}, \quad t \geq 0. \tag{3}$$

2. The density of the Weibull law

$$f(t/\alpha, \beta) = \beta\alpha^{-\beta} t^{\beta - 1} e^{-(\frac{t}{\alpha})^\beta}, \ t \geq 0. \tag{4}$$

3. The density of the truncated logistic law

$$f(t) = \frac{2}{s} e^{\frac{t-\mu}{\sigma}} \left(1 + e^{\frac{t-\mu}{\sigma}}\right)^{-2}, \ t \geq \mu. \quad (5)$$

4. The density of the mixture (p Lognormal and $(1-p)$ Pareto) law

$$f(t) = p\left(\frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{(\log(t)-\mu)^2}{2\sigma^2}}\right) + (1-p)\left((t-c)^{-(\rho+1)}\rho\lambda^{\rho}\right), \ t \geq 0. \quad (6)$$

5. The density of the exponential law

$$f(t) = \frac{1}{\mu} e^{-t/\mu}, \ t \geq 0. \quad (7)$$

In general, these test distributions can be categorized as light (Weibull), medium (Lognormal) and heavy-tailed (Pareto) [3]. Another classification of heavy-tailed distributions can be found in [10], where the above distributions are defined to depend on their parameters, that is to say, they may be either in the class of heavy-tailed, light-tailed or medium-tailed distributions, and this according to their parameters.

Table 1. Different simulated distributions

Mean	Exp λ	LogNormal (a, b)	Weibull (a, b)	Logistic (μ, s)	Mixture: $p * LogN + (1-p)Pareto$ $(p, a, b, \alpha, \beta, c)$
2.00	2.00	(0.5816 , 0.4724)	(2.2397 , 3)	(1.0000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 2.1111)
2.10	2.10	(0.6398 , 0.4521)	(2.3517 , 3)	(1.1000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 2.3333)
2.20	2.20	(0.6945 , 0.4334)	(2.4637 , 3)	(1.2000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 2.5556)
2.30	2.30	(0.7463 , 0.4161)	(2.5756 , 3)	(1.3000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 2.7778)
2.40	2.40	(0.7954 , 0.4001)	(2.6876 , 3)	(1.4000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 3.0000)
2.50	2.50	(0.8421 , 0.3853)	(2.7996 , 3)	(1.5000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 3.2222)
2.60	2.60	(0.8865 , 0.3714)	(2.9116 , 3)	(1.6000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 3.4444)
2.70	2.70	(0.9290 , 0.3585)	(3.0236 , 3)	(1.7000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 3.6667)
2.80	2.80	(0.9696 , 0.3465)	(3.1356 , 3)	(1.8000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 3.8889)
2.90	2.90	(1.0085 , 0.3352)	(3.2476 , 3)	(1.9000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 4.1111)
3.00	3.00	(1.0459 , 0.3246)	(3.3595 , 3)	(2.0000 , 0.7213)	(0.7000 , 0.3051 , 0.4480 , 0 , 3.0000 , 4.3333)

3.3 Numerical and Graphical Results

This section is devoted to present the different numerical and graphical results obtained when studying the influence of heavy-tailed distributions on the stability of a risk model, by considering the distributions defined in the section above.

Table 2. Stability intervals regarding to different distributions

ϵ	Mean	Exp	Lognormal	Weibull	Logistic	Mixture
-0.5	2.00	[0.0002 , 0.3083]	]0 ,0.2955]	[0.0005 , 0.2685]	[0.0003 , 0.1933]	[0.0002 , 0.3002]
-0.4	2.10	[0.0002 , 0.3320]	]0 ,0.3726]	[0.0004 , 0.3424]	[0.0002 , 0.2807]	[0.0002 , 0.3238]
-0.3	2.20	[0.0001 , 0.3610]	]0 ,0.4645]	[0.0003 , 0.431]	[0.0002 , 0.3873]	[0.0002 , 0.3543]
-0.2	2.30	[0.0001 , 0.3999]	]0 ,0.5826]	[0.0003 , 0.547]	[0.0002 , 0.5264]	[0.0001 , 0.3965]
-0.1	2.40	[0.0001 , 0.4627]	]0 ,0.7565]	[0.0002 , 0.7306]	[0.0001 , 0.7357]	[0.0001 , 0.4657]
0.00	2.50	] 0 , ∞ [	]0 , ∞ [	] 0 , ∞ [	] 0 , ∞ [	] 0 , ∞ [
+0.1	2.60	[0.0001 , 0.6172]	] 0 , 0.7571]	[0.0002 , 0.7121]	[0.0001 , 0.7166]	[0.0001 , 0.5786]
+0.2	2.70	[0.0001 , 0.5295]	] 0 , 0.5590]	[0.0002 , 0.5261]	[0.0002 , 0.4921]	[0.0001 , 0.4722]
+0.3	2.80	[0.0001 , 0.4772]	] 0 , 0.4330]	[0.0003 , 0.4145]	[0.0002 , 0.3465]	[0.0002 , 0.4066]
+0.4	2.90	[0.0002 , 0.4398]	] 0 , 0.3399]	[0.0004 , 0.3352]	[0.0002 , 0.2397]	[0.0002 , 0.3591]
+0.5	3.00	[0.0002 , 0.4108]	] 0 , 0.2663]	[0.0004 , 0.2744]	[0.0003 , 0.1573]	[0.0002 , 0.3224]

Table 3. Stability bound Γ regarding to different distributions

ϵ	Mean	Exp	Lognormal	Weibull	Logistic	Mixture
-0.5	2.00	0.1954	1.0098	0.9509	2.0178	0.2286
-0.4	2.10	0.1463	0.6713	0.6224	1.1851	0.1823
-0.3	2.20	0.1032	0.4302	0.3943	0.6986	0.1361
-0.2	2.30	0.0649	0.2498	0.2273	0.3819	0.0903
-0.1	2.40	0.0307	0.1105	0.0999	0.1612	0.0449
0.00	2.50	0	0	0	0	0
+0.1	2.60	0.0267	0.1087	0.0957	0.1613	0.0427
+0.2	2.70	0.0534	0.2418	0.2064	0.3819	0.0853
+0.3	2.80	0.0801	0.4068	0.3357	0.6990	0.1277
+0.4	2.90	0.1067	0.6166	0.4883	1.1861	0.1695
+0.5	3.00	0.1333	0.8915	0.6704	2.0216	0.2106

3.4 Discussion of Results

Note, according to Table 2, that for all the distributions, the stability domain decreases with the increase of the perturbation ϵ. It is evident that a risk model tends to not be stable with a great perturbation. Note also the closure of the stability domains of the mixture distribution to those of the exponential one.

Notice also, following Table 3 and Figure 1, that the strong stability bound Γ increases with the increase of the perturbation ϵ. Even taking distributions having the same mean as the exponential one, one obtains bounds relatively far away from those of the exponential one. This can be explained by the influence of the weight of the tails of the different considered distributions. Comparing to the other distributions, we note that the strong stability bound for the mixture distribution is more closer to that of the exponential one. May be it is due to the special choice of the parameters of this distribution. That is to say, one may be able, in this case, to justify the approximation of the risk model with a general mixture claim distribution by another risk model governed by an exponential law.

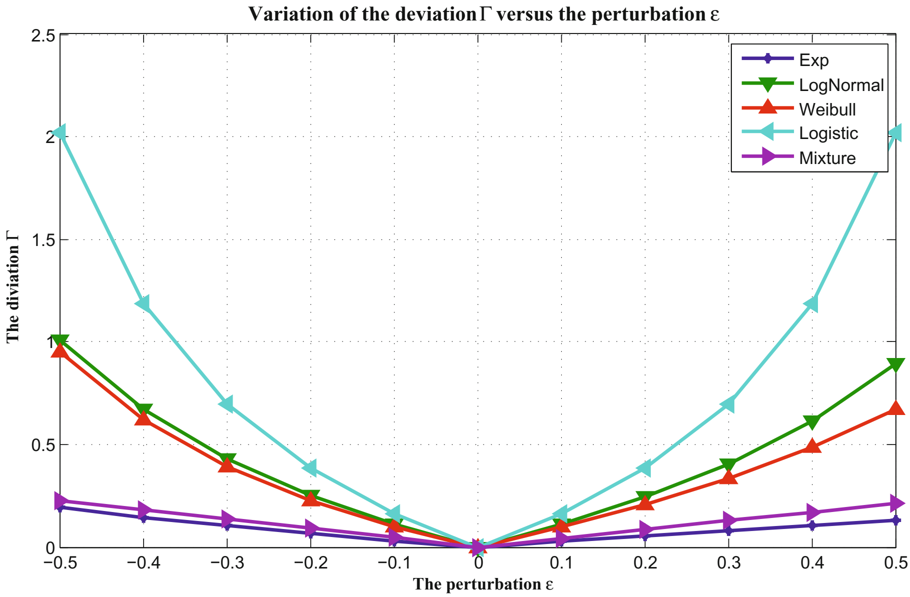

Fig. 1. Variation of the stability bound Γ regarding to different distributions

Note that in the literature, many authors pointed out the limits of the results of Kalashnikov [7] on the stability of risk models and the difficulty of applying them in case of large claims (heavy-tailed distributions). The present results show that in some situations, approximating the characteristics of a risk model with a general heavy-tailed distribution by a classical model is possible, that is to say, one may approach its characteristics by those of a model governed by an exponential distribution (see Tables 2 and 3 and Figure 1). This approximation is in connection not only with the weight of the tail but also with other criteria such as: the shape of the distribution, dispersion parameter, ...

4 Conclusion

We are interested, in this work, in the approximation of the ruin probability of a classical risk model by the strong stability method. We studied the impact of some large claims (heavy-tailed distributions) on the quality of this approximation. A comparative study based on numerical examples and simulation results, involving different heavy-tailed distributions, is performed.

The literature indicates that, in general, the results of Kalashnikov [7] on the stability of risk models, are not applicable for heavy-tailed distributions. The present results show that, in some situations, the approximation of the characteristics of a risk model with a heavy-tailed distribution by a classical model (with an exponential law) is possible. This approximation is linked not only with the weight of the tail but also with other criteria such as the shape of the distribution. These results could be very useful in the case of an unknown distribution that must be replaced by an estimate (kernel estimate). Indeed, in this case, we need a prior knowledge, at least approximately, of the shape of the unknown distribution.

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Author Index

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