

# Tissue Segmentation Based on Two-Way ANOVA and Gram-Schmidt Orthogonal Polynomial Approximation

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## Abstract

This paper presents a method for segmenting tissues in images, regardless of their type or the nature of the material—whether from magnetic resonance imaging (MRI) or other sources. The approach relies on two numerical techniques: orthogonal polynomial approximation and the two-way ANOVA (anova2) statistical method. The study involves two types of images: a normal (healthy) image without impurities or tumors, and a deformed image, either artificially modified or representing a brain with a tumor. By applying both methods, tissue structures can be distinguished and enhanced, providing researchers and medical professionals with clearer image interpretations in a shorter time. The results demonstrate that while the anova2 method offers higher accuracy for material images, the orthogonal polynomial approximation proves more effective and faster for medical images.

## Keywords

Tissue segmentation; MRI; orthogonal polynomial approximation; Gram-Schmidt; two-way ANOVA.

## 1. Introduction

One of the primary objectives of numerical analysis is to identify and develop algorithms that can be efficiently implemented on computers to solve mathematical problems that lack exact solutions [1]. Such algorithms are designed to provide accurate approximations, and once obtained, numerical analysis offers well-established techniques for estimating the associated error bounds [2][3]. This dual focus on approximation and error control has made numerical methods indispensable in science and engineering.

Over the decades, numerical analysis has found applications in nearly every domain, including engineering, physical sciences, life sciences, medicine, social sciences, architecture, business, and even the arts [4][5]. Typical uses include approximation, simulation, and estimation tasks, where analytical solutions are either difficult or impossible to compute [6]. In the context of image processing, numerical methods have been successfully applied to tasks such as segmentation, enhancement, and pattern recognition, enabling faster and more reliable analyses of complex visual data [7][8].

Recent studies have highlighted the potential of combining statistical and mathematical approaches for medical image analysis, particularly for tissue segmentation [9][10]. Building on this foundation, our paper focuses on two specific numerical techniques: two-way analysis of variance (ANOVA2) and orthogonal polynomial approximation using the Gram-Schmidt process. The former provides a statistical framework for analyzing variations across image data, while the latter offers a mathematical approach to approximate image structures with orthogonal polynomials. We apply these methods to process images from various domains, including medical imaging, to display tissue structures with clarity and precision, while aiming for reduced computation time.

The remainder of this paper is structured as follows: Section 2 reviews the theoretical background of the methods; Section 3 details the implementation and dataset; Section 4 presents results and analysis; and Section 5 provides conclusions and recommendations for future work.

## 2. Background of the Methods

In this section, we present the theoretical basis of the two numerical techniques applied in this study: orthogonal polynomial approximation using the Gram-Schmidt process, and the two-way analysis of variance (ANOVA2). Both methods are well-established tools in numerical analysis and have valuable applications in image processing.

### 2.1. Orthogonal Polynomial Approximation and Gram-Schmidt

Let  $\{\phi_0(x), \phi_1(x), \dots, \phi_n(x)\}$  denote an orthogonal set of polynomials over a domain  $D$  with respect to a weight function  $w(x)$ . The inner product is defined as:

$$\langle \phi_i, \phi_j \rangle = \int \phi_i(x) \phi_j(x) w(x) dx \quad (1)$$

Two functions are orthogonal if  $\langle \phi_i, \phi_j \rangle = 0$  when  $i \neq j$ .

The Gram-Schmidt process generates this orthogonal basis from a set of linearly independent functions  $\{1, x, x^2, \dots\}$  as:

$$\phi_k(x) = x^k - \sum_{j=0}^{k-1} \frac{\langle x^k, \phi_j \rangle}{\langle \phi_j, \phi_j \rangle} \phi_j(x), \quad k = 1, 2, \dots, n \quad (2)$$

The orthogonal polynomial approximation of a function  $f(x)$  is then:

$$f(x) \approx \sum_{i=0}^n c_i \phi_i(x) \quad (3)$$

Where:  $c_i = \frac{\langle f, \phi_i \rangle}{\langle \phi_i, \phi_i \rangle}$

In tissue segmentation,  $f(x)$  represents the intensity profile of an image slice or region. The orthogonal expansion helps to smooth noise and emphasize structural features.

### 2.2. Two-Way Analysis of Variance (ANOVA2)

Analysis of variance (ANOVA) and also called the Fisher analysis of variance, and it is the extension of the t- and z-tests. The term became well known in 1925, after appearing in Fisher's book, "Statistical Methods for Research Workers."<sup>3</sup> It was employed in experimental psychology and later expanded to subjects that were more complex.

It is an analysis tool used in statistics that splits an observed aggregate variability found inside a data set into two parts: systematic factors and random factors. The systematic factors have a statistical influence on the given data set, while the random factors do not. Analysts use the ANOVA test to determine the influence that independent variables have on the dependent variable in a regression study<sup>[3]</sup>.

ANOVA2 analyzes the effect of two independent categorical factors A and B on a continuous variable Y. The model is:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \quad (4)$$

where:

- $\mu$  is the overall mean
- $\alpha_i$  is the effect of the  $i^{th}$  level of factor A
- $\beta_j$  is the effect of the  $j^{th}$  level of factor B
- $(\alpha\beta)_{ij}$  is the interaction effect
- $\epsilon_{ijk}$  is the random error

The sums of squares are computed as:

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E \tag{5}$$

where:

- $SS_A$  is the sum of squares for factor A
- $SS_B$  is the sum of squares for factor B
- $SS_{AB}$  is the interaction sum of squares
- $SS_E$  is the error sum of squares

Each source of variation is tested with:

$$F = \frac{MS_{source}}{MS_E} \tag{6}$$

Where MS indicates Mean Squares (sum of squares divided by degrees of freedom).

In tissue segmentation, factors A and B can correspond to pixel regions or features (e.g., row and column blocks), and ANOVA2 tests whether tissue types produce statistically significant differences in pixel intensity distributions.

### 2.3. Pre-processing

Before applying the main segmentation algorithms, a pre-processing phase is performed to prepare the images and ensure accurate tissue segmentation. This phase involves reading the input images and applying either the two-way analysis of variance (ANOVA2) or orthogonal polynomial approximation using the Gram-Schmidt (GS) process, depending on the selected method.

The dataset used in this work includes a variety of images collected from different sources:

- **Material images** from the research center of CDTA (Centre de Développement des Technologies Avancées) in Algiers, Algeria.
- **Medical MRI images** obtained from the MRI service of the private clinic of Professor Yaker, located in Old Kouba, Algiers.
- **Additional reference images** collected from public datasets and scientific resources available on the Internet.

Each algorithm follows a structured sequence of steps. Table 1 summarizes these steps for both methods:

Table 1. Summary of the main steps for ANOVA2 and Gram-Schmidt-based tissue segmentation algorithms.

Step	ANOVA2 Algorithm	Gram-Schmidt Algorithm
<b>1. Input</b>	Read image 1 (reference) and image 2 (modified or pathological).	Read image 1 (reference) and image 2 (modified or pathological).
<b>2. Method applied</b>	Apply two-way ANOVA (ANOVA2) to image data.	Apply orthogonal polynomial approximation using Gram-Schmidt process.
<b>3. Key computation</b>	Calculate F-statistics for factors and their interaction.	Compute approximate orthogonal polynomials $\phi_k(x)$
<b>4. Projection</b>	Project statistical variation results onto the modified image.	Project polynomial approximation onto the modified image.
<b>5. Output / Display</b>	Display segmented or highlighted tissue regions based on ANOVA2.	Display segmented or smoothed tissue regions based on approximation.

In the ANOVA2-based method, statistical tests are applied to determine the significance of intensity variations caused by different factors (such as row and column regions of the image), and the results are projected onto the image to highlight significant tissue differences. The F-statistic values help in detecting boundaries or anomalies that differ significantly from normal patterns.

In the Gram-Schmidt-based method, the image intensity data is approximated using orthogonal polynomials generated by the Gram-Schmidt process. The resulting polynomial projection smooths the image and emphasizes underlying tissue structures, enabling clearer segmentation of regions with different properties.

Both pre-processing workflows aim to enhance the visibility and interpretability of tissues within the images, facilitating reliable segmentation in the subsequent processing stages.

### 2.4. Results and discussion

For each example presented in this study, we work with two primary input images:

Image 1: The original or reference image representing a normal, healthy case without pathological features.

Image 2: The corresponding modified or pathological image representing tissue alterations, deformations, or anomalies.

a) **Example 1:** MRI Brain Image

The next images obtained from clinical service of MRI in old kouba –Algiers. The first one is for present normal and the second is for sick patient (pathological image).

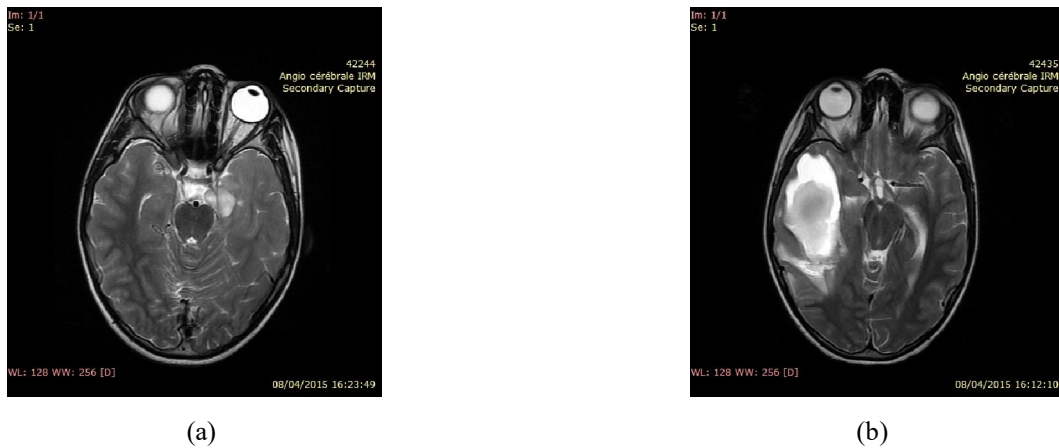


Fig. 1. Original Images (a) Image-1 (b) Image-2

When the ANOVA2 method was applied to the pathological image, the segmentation performance was found to be suboptimal. As shown in Fig.2 (c-d), the accuracy achieved was only 16.14%, with a computation time of 16.36 seconds. This indicates that ANOVA2 struggled to effectively distinguish the tissue structures in this particular medical image.

Additionally, the histogram illustrated in Fig.2 (b) reveals that the image intensity levels are primarily distributed between 0.1 and 0.5, and do not provide a clear separation of all tissue types present within the image. This further supports the observation that ANOVA2, in this case, was unable to achieve satisfactory tissue segmentation.

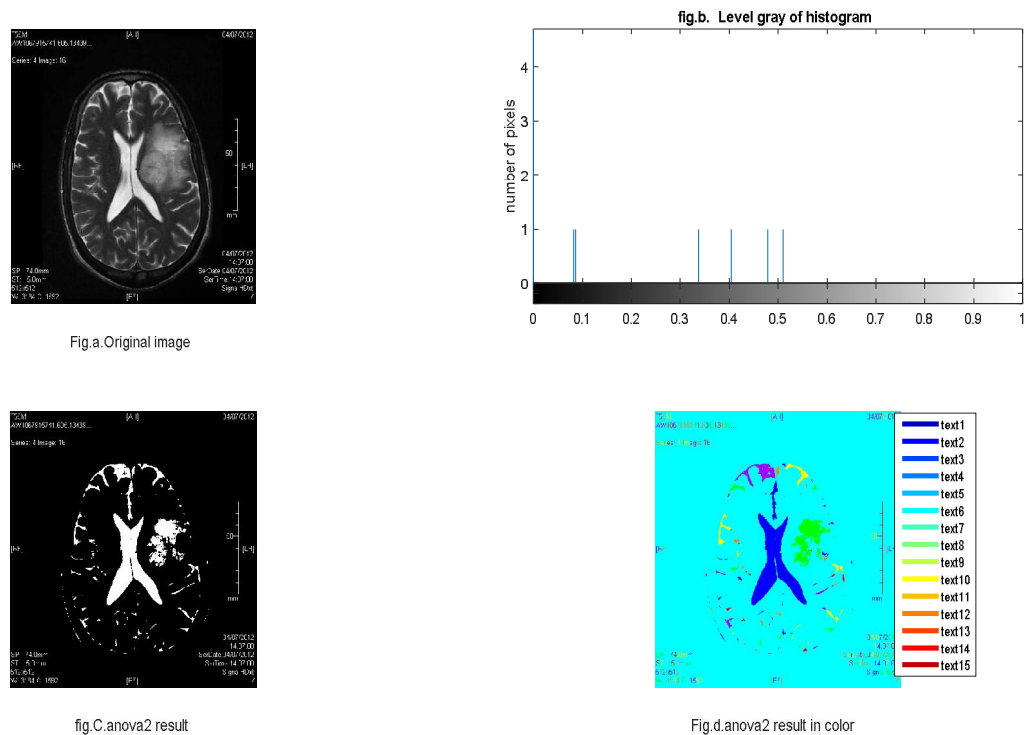
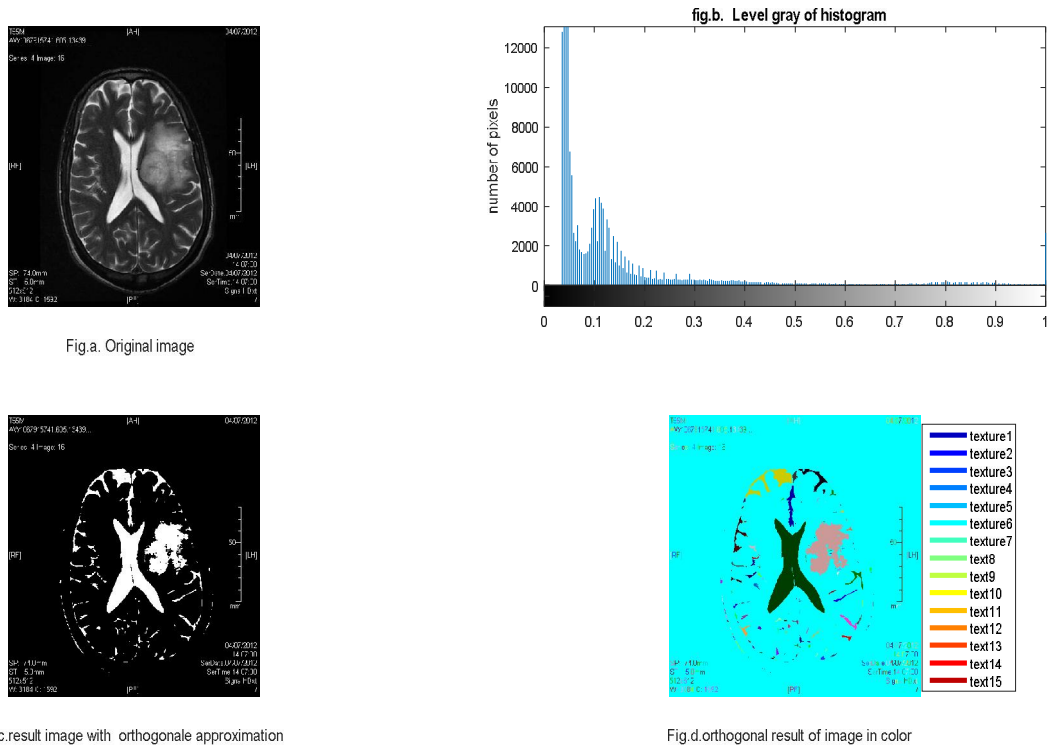


Fig. 2. Results (a) the original MRI image (b) gray level of the image (c) Applied Anova2 result (d) Colored Anova2 result

The next method applied is the Gram-Schmidt orthogonal polynomial approximation, which demonstrated significantly better performance in segmenting the tissues compared to ANOVA2. When applied to the same pathological image, the Gram-Schmidt method achieved an accuracy of 75.5% with a reduced computation time of 5.09 seconds.

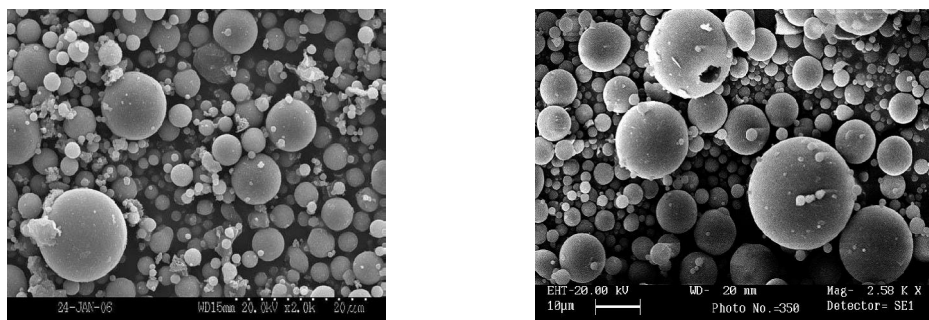


**Fig. 3.** Results (a) the original MRI image (b) gray level of the MRI image (c) Applied OA result (d) Colored OA result

The histogram of the processed image, shown in Fig.3(b), clearly illustrates the distribution of gray levels corresponding to different tissues. The intensity levels are well defined between 0.1 and 0.5, enabling easier distinction of tissue structures within the image. This confirms the superiority of Gram-Schmidt approximation for this type of medical image segmentation.

**b) Example 2:** scanning electron microscopy (SEM) of coal fly ash

The fourth example is an image obtained from the Internet, representing scanning electron microscopy (SEM) of coal fly ash see fig.4. This type of image is commonly used in material sciences to analyze particle morphology and distribution.



(a) (b)  
**Fig. 4.** Original SEM Images (a) Image-1 (b) Image-2

Both the ANOVA2 and Gram-Schmidt methods were applied to this example. The results showed that, consistent with the previous cases, Gram-Schmidt orthogonal polynomial approximation provided a clearer

segmentation of particle boundaries, with improved accuracy and reduced computation time compared to ANOVA2.

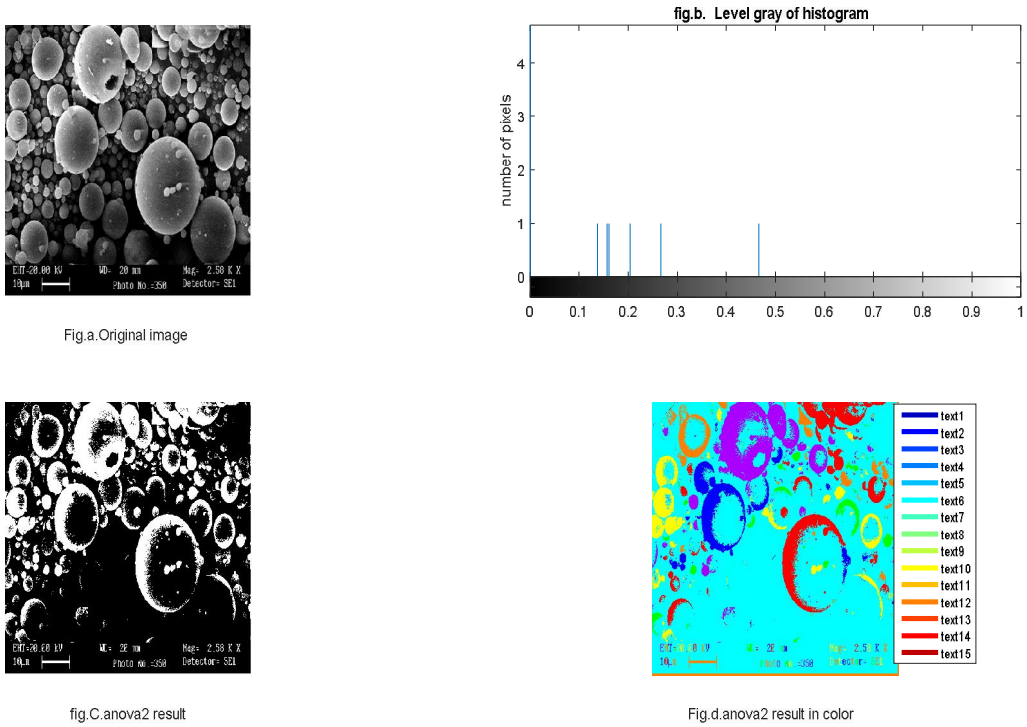


Fig. 5. Results (a) the original SEM image (b) gray level of the image (c) Applied Anova2 result (d) Colored Anova2 result

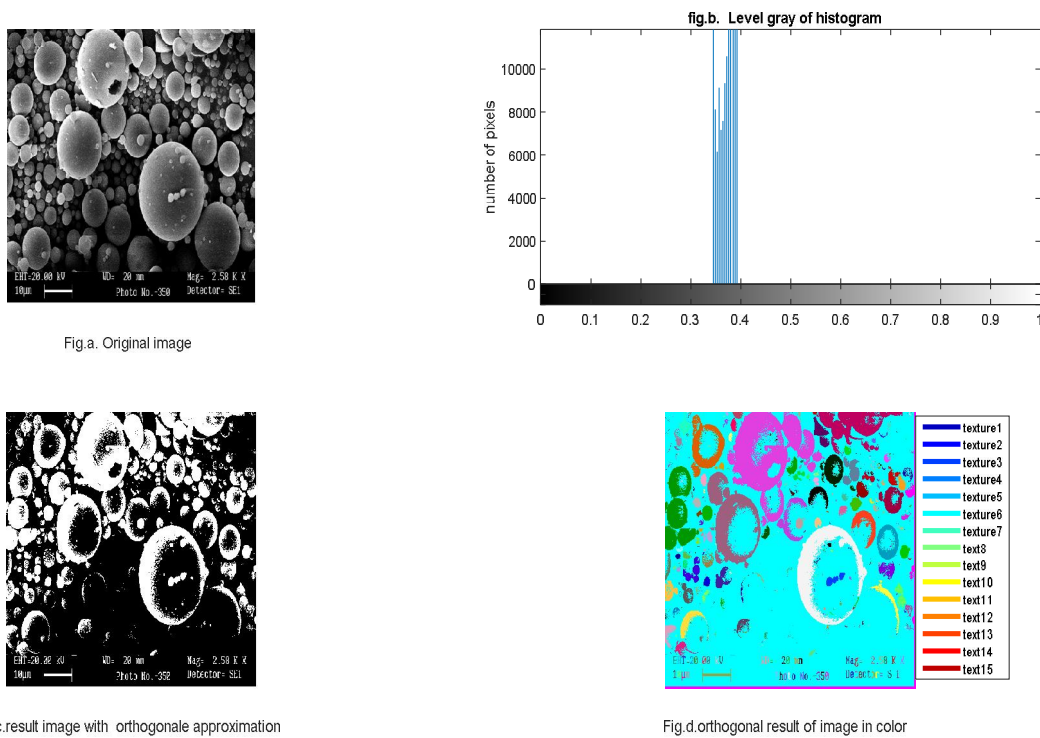


Fig. 6. Results (a) the original SEM image (b) gray level of the SEM image (c) Applied OA result (d) Colored OA result

The segmentation results for the SEM image of coal fly ash further confirmed the limitations of both methods in this specific material imaging case. When the ANOVA2 method was applied, the segmentation result, shown in

Fig.5 (c-d), produced an accuracy of only 40.60% with an execution time of 0.54 seconds. Similarly, the Gram-Schmidt orthogonal polynomial approximation, as shown in Fig.6, resulted in a segmentation accuracy of 37.62%, with a longer elapsed time of 7.76 seconds. The histogram of the segmented image indicates that the pixel intensities corresponding to the detected particles or regions mainly fall within the gray level range of 0.3 to 0.4, which did not provide sufficient contrast for clearly distinguishing all particle boundaries.

c) **Example 3:** The final example analyzed in this study is an image obtained from the Internet, specifically from a resource on scanning electron microscopy (SEM) [11]. This example clearly demonstrates the advantage of the Gram-Schmidt method over ANOVA2.

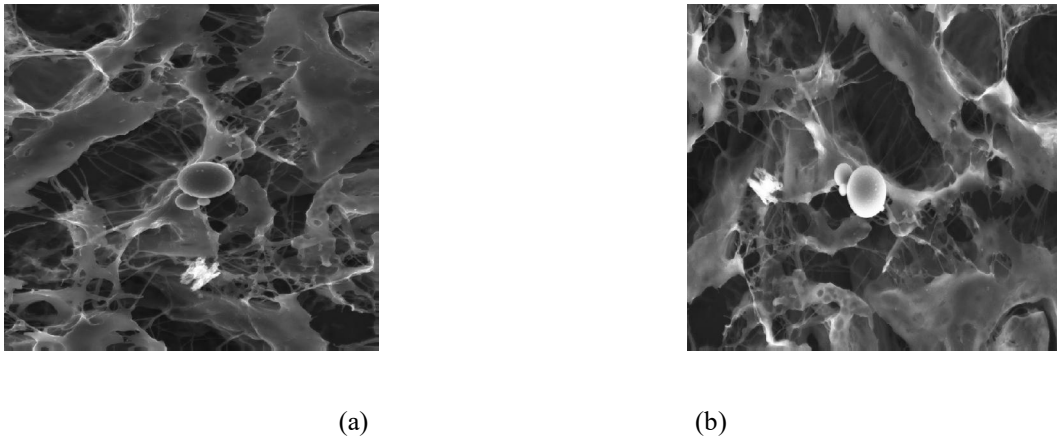


Fig. 7. Original SEM Images (a) Image-1 (b) Image-2

When applying the ANOVA2 method, as shown in Fig.8, the segmentation result was clearly insufficient for reliable tissue detection, with a low accuracy of 24.8% and an elapsed time of 21.53 seconds. The segmentation output revealed incomplete boundary detection, and finer tissue details were often missed or merged.

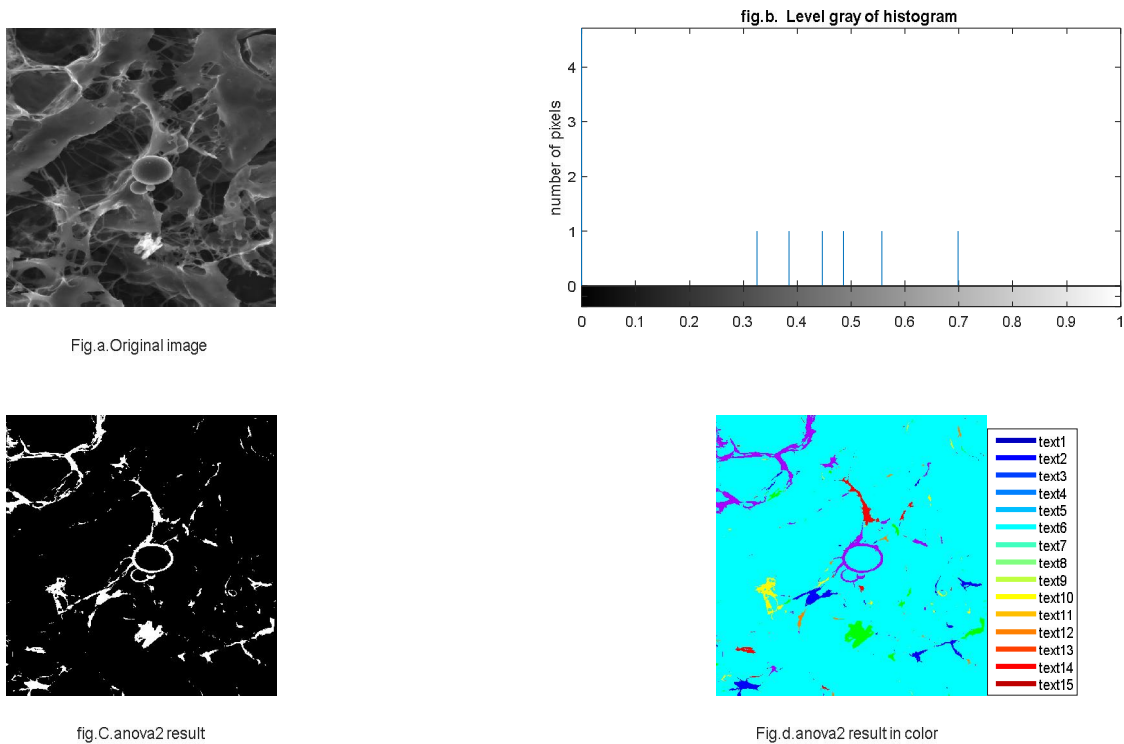
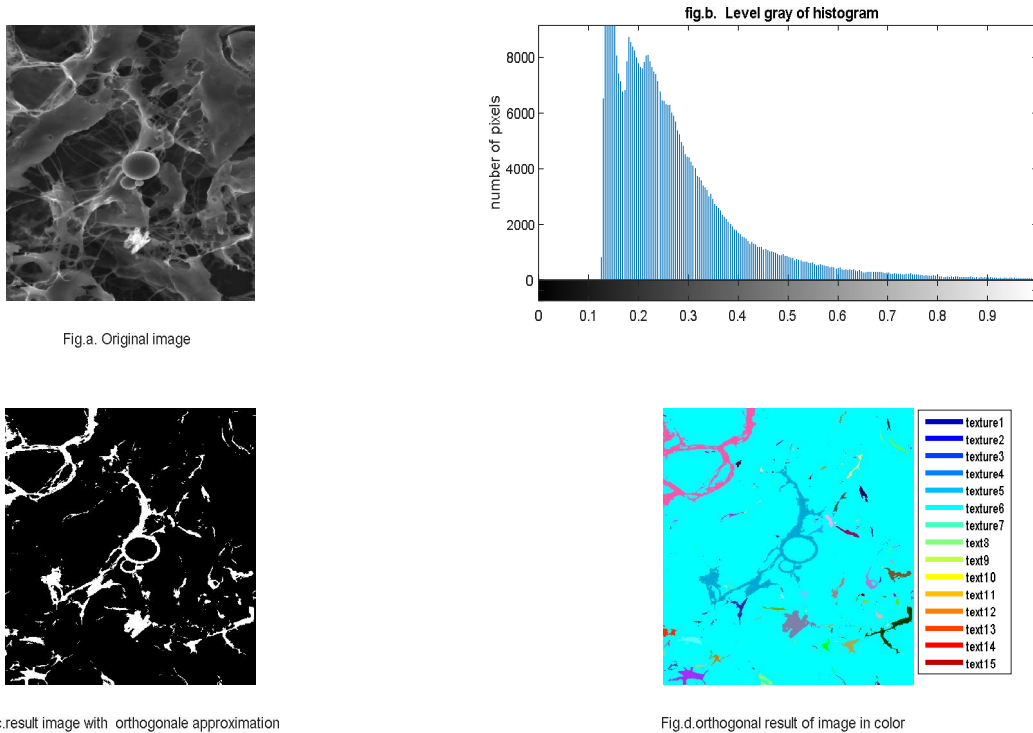


Fig. 8. Results (a) the original SEM image (b) gray level of the image (c) Applied Anova2 result (d) Colored Anova2 result



**Fig. 6.** Results (a) the original SEM image (b) gray level of the SEM image (c) Applied OA result (d) Colored OA result

In contrast, the Gram-Schmidt method, illustrated in Fig.9, achieved excellent segmentation, with an accuracy of 99.27% and a faster processing time of 17.29 seconds. The histogram of the Gram-Schmidt result confirms clear separation of tissue structures across distinct gray-level ranges, highlighting the method's effectiveness in preserving structural integrity during segmentation.

These results in Table.2 highlight the superior precision and reliability of the Gram-Schmidt method in detecting tissues and potential lesions in certain cases, underscoring its promise as an effective tool for medical image analysis. This work lays the foundation for future research, including exploring orthogonality principles within ANOVA2 to enhance segmentation accuracy further.

**Table 2.** ANOVA2 vs. Gram-Schmidt accuracy

Example	ANOVA2 Accuracy (%)	Gram-Schmidt Accuracy (%)
1	16.14	75.54
2	40.60	37.62
3	24.80	99.27

#### 4. Conclusion

In this paper, we addressed the important challenge of tissue segmentation in medical and experimental images by applying and evaluating two complementary statistical approaches: Two-Way Analysis of Variance (ANOVA2) and the sum of squares approximation using Gram-Schmidt orthogonal polynomial techniques. The goal was to explore and compare these methods in terms of their effectiveness in accurately segmenting and identifying tissues, including impurities or lesions that could otherwise remain undetected by conventional analysis. Through a series of experiments focusing on selected image examples, our comparative analysis demonstrated the valuable roles that both methods can play in this domain. The results clearly indicated that, while each method has its strengths, the Gram-Schmidt orthogonal polynomial approximation consistently provided higher precision than ANOVA2 in most of the tested cases, particularly in examples where traditional variance-based techniques were less effective. This superior performance highlights the potential of Gram-Schmidt as a robust tool for improving tissue segmentation outcomes. The findings of this work offer meaningful insights for researchers who are developing or refining segmentation techniques, as well as for medical

professionals seeking enhanced tools for analyzing medical images and detecting tissue anomalies. By providing clearer and more precise identification of different tissue structures, including impurities and lesions, our approach may contribute to more accurate diagnoses and more reliable experimental results.

As a direction for future research, we propose a deeper investigation into the integration of orthogonality concepts within the ANOVA2 framework. Such an enhancement could combine the strengths of both methods and potentially lead to more powerful and versatile segmentation techniques capable of addressing a wider range of image analysis challenges. We believe that this line of inquiry holds promise for advancing both the theoretical understanding and practical applications of statistical methods in medical image segmentation.

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## Biography

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