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DNLR Approach of the Behavior and Damage of Thermoplastic Polymers (HDPE)

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Abstract

This study proposes a novel formulation of the Distribution of Non-Linear Relaxation (DNLR) model to accurately capture the mechanical behavior and damage evolution of semi-crystalline thermoplastic polymers, specifically high-density polyethylene (HDPE), under large uniaxial deformations. Building upon the generalized Gibbs relationship for out-of-equilibrium systems, the proposed approach integrates an effective elastic modulus to account for damage-induced stiffness degradation. The model uses an empirical constitutive law to describe the evolution of the Young's modulus as a function of axial strain, coupled with a statistical hyperelastic framework to define the equilibrium stress response. Numerical simulations reveal that the apparent elastic modulus initially decreases significantly due to micro-cavitation damage, followed by a continuous recovery phase driven by macromolecular chain reorientation until ultimate failure. The predicted stress-strain curves and modulus evolution demonstrate strong agreement with experimental tensile data across the entire deformation range. These findings confirm that the enhanced DNLR formulation effectively bridges dissipative thermodynamics with microscopic deformation mechanisms, providing a robust predictive tool for modeling damage progression and non-linear viscoelastic behavior in thermoplastic and biopolymer systems. The model's capacity to quantitatively reproduce high-deformation experimental responses highlights its potential for advanced material characterization and structural integrity assessment.

Keywords: *Constitutive modeling, Damage evolution, High-density polyethylene, Irreversible thermodynamics, Non-linear relaxation.*

I. Introduction

The plastic deformation mechanisms in semi-crystalline High-Density Polyethylene (HDPE[1]) are governed by complex interactions between its amorphous and crystalline phases [1], [2]. At small deformations, interlamellar sliding and the rotation of crystalline stacks predominate, while increased stress activates primary crystallographic mechanisms such as chain-direction and perpendicular slip, occasionally accompanied by twinning or martensitic transformations [1], [3]. As deformation progresses, the original lamellar structure undergoes fragmentation and micronecking, reorganizing into a fibrillar structure interconnected by highly stretched tie molecules, which induces hyperelastic hardening [1], [2], [3]. Concurrently, this structural evolution is intrinsically linked to damage mechanisms, notably cavitation and microvoid formation near the yield point, which accelerate lamellar fragmentation as the material approaches failure [1].

HDPE is a semi-crystalline polymer, its crystallization result from the regular piling of macromolecular chains. Their extremities are different from the rest of the polymer, these macromolecules cannot be fully regular. As a result, the existence of both amorphous and crystalline phases [1]. This microstructure is described at first by the fringe and micelles model [4], in which the crystalline zones are represented by ordered sequences where the chains are aligned. The size of the crystalline zones varies between 5 and 50 nm. In considering their length, the macromolecules can be parts of many crystals, which are guided at random. We can distinguish two extreme configurations related to the folding of macromolecular chains: tight arrangement and disordered arrangement [3], [4]. In this last case, the same chain can then participate at the same time to the amorphous and the crystalline phases [5] contrary to the first mode of folding where the chain participates only to the crystallite formation [4].

One of the most remarkable characteristics of semi-crystalline thermoplastics is their capacity to undergo a large plastic deformation before the break at room temperature [1], [4]. This phenomenon results partially because their glassy

[1] HDPE : High Density Polyethylene

transition temperature (-100°C for PE[2]) is well situated below the ambient temperature [1], [6]. Contrary to the glassy polymers for which the deformation is located in the fine bands of cutting, for the semi-crystalline polymers the plastic deformation intervenes in a macroscopic and not homogeneous way in tests of uniaxial drive [7], [8]. This phenomenon of deformation localization, called striction or plastic instability, was widely studied [1], [3], [5], [9].

To describe an efficient behaviour law for this material, we recourse to a model using the approach developed by Cunat [10], [11], based on the Thermodynamics of the Irreversible Processes (TPI) to the scale of the RVE[3] [1], [6]. This approach leans on Gibbs relationship generalization in the case where the local equilibrium balance is broken [6], [11], [12]. It defines itself like an extension of the concepts initially introduced by De Donder [11], [13], [14], the local dissipation can be considered physically as the result of definite internal reorganizations assimilated to non-stochiometric chemical reactions; these processes are characterized then by their degrees of advancement and their affinities [1], [6]. The idea is to achieve a modal description in presence of non-linearities [11], [15]. This approach was used with success these last years, to simulate elastic-viscoplastic polymers' behaviour. The beginning works were especially focused on the definition and on the role of the relaxed state and the spectre of the time relaxation in the modelling. Several writings were proposed, we shall evoke only that of Loukil, M'rabet and Arieby concerning the mechanical behaviour modelling of the semi-crystalline polymers, particularly HDPE.

In the present work, we remind firstly the theoretical framework and the rheological model, which is adopted and secondly, we suggest a new formulation of the D.N.L.R[4]. model, which considers the damaging effects.

II. Material and methods

II.1 Material Specification and Specimen Characteristics

The material investigated in this study consists of high-density polyethylene (HDPE), specifically PE100 grade granules supplied by the CHIALI company (Sidi Bel Abbes, Algeria) [16]. This semi-crystalline thermoplastic is characterized by a molar mass of approximately 300,000 g/mol and exhibits a structural morphology composed of spherulitic aggregations of amorphous and crystalline phases. Differential scanning calorimetry (DSC) characterization established the melting temperature (T_m) at 135°C and the glass transition temperature (T_g) at -125°C [16].

II.2 Uniaxial Tensile Testing Configuration

Mechanical characterization was performed using a universal servo hydraulic tensile testing apparatus (MTS systems) under strict isothermal conditions at 25°C [15], [16]. To accurately capture the material's true stress-strain response and damage progression, all tests were conducted under local strain-rate control at a constant rate of $\dot{\varepsilon} = 10^{-3}s^{-1}$.

The local deformations within a Representative Volume Element (RVE) were monitored in real-time using the VideoTraction system [8], [15]. This intelligent optical extensometer utilizes a digital camera to analyze the relative displacements of dot markers printed on the specimen surface [8], [15]. This technique enables the real-time calculation of true (rational) axial strain and true stress, the latter accounting for the instantaneous reduction in cross-sectional area during drawing and localized necking [8], [15]. The experimental data thus obtained, specifically the Young's modulus, yield stress, and hardening behavior, served as the primary calibration set for identifying the numerical parameters of the proposed DNLR model.

III. Theoretical package

III.1 The D.N.L.R. approach and Constitutive equations

The DNLR approach has been initially proposed by Cunat in 1988 [9] for simulating the behaviour of liquid during a decreasing temperature. It is based on a statistical mechanics considering that each "particle" of the material has its own dynamics for its motion degree of freedom (vibration, translation or even rotation). The word ''particle'' means elementary particle (atomic size) or group of particles (cluster), which can be treated with a specific dynamics. The evolution of the corresponding population results from the equilibrium condition by assuming that there is no excess term for the mixing of several particles. Hence, it is possible to build the macroscopic response and to obtain the specific heat as well as the enthalpy or the entropy. This corresponds to the equilibrium or relaxed state. In the case of a sudden quench of a liquid, Cunat assumes that it appears as an excess term of energy or entropy associated with an activity coefficient for each particle. This term plays the role of the composition adaptation in order to establish a partial equilibrium or a constrained equilibrium. The obtained deviation from the equilibrium is just the fluctuation used in the DNLR theory. Due to practical reasons, the specificity of the DNLR is to choose a modal base. The principal advantage of this approach is based on

[2] PE : Polyethylene
[3] RVE : Representative Volume Element
[4] D.N.L.R : Distribution of Non-Linear Relaxations

the introduction of a quasi-infinite number of dissipative modes using only two adjustable parameters for describing the corresponding relaxation spectrum by means of fluctuation theory.

The developed constitutive laws aim to describe the mechanical behaviour of material based on two principle foundations expressed in a representative volume element (RVE):

1. The first one consists to admit that even in out-of-equilibrium condition, the internal energy remains as an extensive quantity and keeps the status of a potential controlled by three independent extensive variables: (i) the entropy (thermal part of the energy), (ii) the volume variations caused by deformation (mechanical part of the energy) and (iii) the whole number of species forming the volume subjected to internal reactions (chemical part of the energy). The potential is a function of two kinds of state variables: internal and observable ones.
2. The second foundation is based on the use of the fluctuation theory to statistically describe the dissipative phenomena in the material.

For each element in a given system, a set of degrees of freedom or internal variables can be used to describe the actual configuration and its dynamic evolution traducing the internal reorganizations. In equilibrium state, these variables are no longer independent and become functions of the controlled variables directly depending on the boundary conditions.

The idea which prevails to the D.N.L.R. formalism consists to spread the common concepts of thermodynamics to situations of non-equilibrium. We admit that the generalized Gibbs relation maintains its status related to a potential function and it contains all the information. It expresses notably the equivalence of the three forms which are related to internal energies, say thermal, mechanical and chemical.

The starting point is the writing of the internal energy as a potential of Gibbs.

$$U = Ts + \sigma e + \sum_k \mu_k \eta_k \ , \qquad (1)$$

where s, e and η_k represent respectively, the specific entropy, the strain vector and the mole number of components $k. T, \sigma$ and μ_k are their associated dual variables representing, respectively, the temperature, the stress vector and the chemical potential.

The different terms in Eq. (1) are defined as the first order partial derivatives of internal energy. At the equilibrium state, they recover their usual signification, i.e., the thermometric value of the temperature for example.

For a regular representative volume element (RVE), the internal energy is expressed according to variables of a thermal and mechanical origin and internal variables of a microstructure origin. We can, then write the Gibbs relation in a more condensed form:

$$\dot{U} = Y \cdot \dot{y} \qquad (2)$$

With $Y = \frac{\partial U}{\partial y} = (T, \sigma, A)$ which represents the generalized force to $y = (s, e, z)$, the vector of extensive variables. The affinity A is the thermodynamic force which is associated to the independent and internal variables z. The principal variation which prevails to the second law of the thermodynamics induces naturally the return of the system in its equilibrium state.

To characterize the local behaviour of the elementary representative volume, we consider the new potential $\Psi = \Psi(\gamma, z)$, where γ and z are, respectively, the vectors of controlled and internal variables. Indeed, the form of the potential is not unique, and the use of the Legendre rules allows modifying the potential with respect to the observable variables at the RVE boundaries. The variation of this potential is adapted to experimental conditions and is written as:

$$d\psi_k = \sum_{m=1}^{q} \beta_m d\gamma_m - \sum_{j=1}^{p} A^j dz^j \ , \qquad (3)$$

where q and p are the dimensions of generalized size γ and z respectively.

The variables β_m and γ_m represent respectively the observable variables and the perturbation of the system. In case of an elastic-plastic behaviour, β corresponds to a stress σ and γ to a strain ε.

After differentiation of eq. (3), one obtains

$$\beta_m\left(\gamma_n, z^j\right) = \frac{\partial \psi_k\left(\gamma_n, z^j\right)}{\partial \gamma_m} \qquad (4)$$

$$A^j\left(\gamma_m, z^j\right) = \frac{\partial \psi_k\left(\gamma_n, z^j\right)}{\partial z^j} \qquad (5)$$

Then:

$$d\beta_m\left(\gamma_n, z^j\right) = \sum_{n=1}^{q} \frac{\partial^2 \psi_k\left(\gamma_n, z^j\right)}{\partial \gamma_m \partial \gamma_n} d\gamma_n + \sum_{j=1}^{p} \frac{\partial^2 \psi_k\left(\gamma_n, z^j\right)}{\partial \gamma_m \partial z^j} dz^j \qquad (6)$$

$$dA^j\left(\gamma_m, z^j\right) = -\sum_{n=1}^{q} \frac{\partial^2 \psi_k\left(\gamma_n, z^j\right)}{\partial z^j \partial \gamma_n} d\gamma_n - \sum_{j=1}^{p} \frac{\partial^2 \psi_k\left(\gamma_m, z^j\right)}{\partial \gamma z^j \partial z^j} dz^j \qquad (7)$$

From which the writing, under the following matrix form:

$$\dot{\sigma} = a^u.\dot{\varepsilon} + \bar{b}.\dot{z} \quad (8)$$

$$-\dot{A} = \bar{b}^t.\dot{\varepsilon} + g.\dot{z}\,, \quad (9)$$

where a^u is a square symmetric tensor of Tisza which is responsible for the instantaneous response of the system and coupling the controlled state variables with themselves. The matrix $\bar{b}$ is a rectangular tensor coupling the reversible state variables with the irreversible ones to describe the dissipation. g is a definite positive square symmetric tensor coupling the internal variables with themselves.

The two constitutive and incremental relations (8) and (9) can be written in a synthetic manner as follows:

$$\begin{pmatrix} \dot{\sigma} \\ -\dot{A} \end{pmatrix} = \begin{pmatrix} a^u & \bar{b} \\ \bar{b}^t & g \end{pmatrix} \begin{pmatrix} \dot{\varepsilon} \\ \dot{z} \end{pmatrix} \quad (10)$$

In a purely mechanical situation:

$$\dot{\sigma} = \sum_{j=1}^{N} \dot{\sigma}_j = \sum_{j=1}^{N} \left(p_0^j a^u \dot{\varepsilon} - \frac{\sigma^j - \sigma^{j,r}}{a(t)\tau_j^r} \right) \quad (11)$$

With

$$\dot{\sigma}^{j,r} = p_0^j a^u \dot{\varepsilon}\,,$$

$$\tau_j^r = \frac{h}{K_B T} \exp\left(\Delta F_j^{+,r}\right),$$

$$a(t) = \exp\left(\frac{K_\sigma \left| \sigma - \sigma^r \right|}{RT} \right).$$

where K_σ is homogeneous to an activation volume. The weight p_0^j of each mode is obtained by an extension of the fluctuation theory which leads to:

$$p_0^j = \frac{\sqrt{\tau^{j,r}}}{\sum_j \sqrt{\tau^{j,r}}} \quad (12)$$

where $\tau^{j,r}$ is the relaxation time of the j^{th} mode. Relaxation times $\tau^{j,r}$ distributed on 3 decades of the time scale according to $\tau^{j,r} = \tau^{max,r} 10^{-3(n-j)/(n-1)}$ where $\tau^{max,r}$ relates to the slowest process and n is the number of relaxation modes.

Among others authors, M.Loukil [14] and K. M'rabat [15] have worked on this model. Loukil described the state of the polymers in large deformations like a state governed by an elastic type law with the help of the Moony-Rivlin model, well enough adapted to the modelling of the elastomers. To characterize the constraint relaxed of the HDPE in large distortions experimentally, M'rabet did several sequences of relaxation from different load points situated in the traction domain. More lately, Arieby introduced the damaging effects in prevention of the behaviour of this material in large deformation.

III.2 Proposed Damage Model

Let's recall the meaningful progress of R. Arieby [16] concerning the characterization and the modelling of mechanical behaviour of the HDPE materials. In our case, we introduce the effective elastic modulus to describe damaging. In fact, contrarily to the R. Arieby model where the stress is measured using the real modulus E, we introduce a stress coupling to the damaging using the effective modulus E^{eff}:

$$\dot{\sigma}_1 = \sum_{j=1}^{N} \dot{\sigma}_1^j = E^{eff}\,\dot{\varepsilon}_1 - \sum_{j=1}^{N} \frac{\sigma_1^j - P_j^r \sigma_1^{j,r}}{\tau_j^r} \quad (13)$$

$$\dot{\sigma}_1^r = \sum_{j=1}^{N} \dot{\sigma}_1^{j,r} = E^{eff,r}\,\dot{\varepsilon}_1 - \sum_{j=1}^{N} \frac{\sigma_1^{j,r} - P_j^{eq} \sigma_1^{j,eq}}{\tau_j^{eq}} \quad (14)$$

To fully account for the dissipative nature of HDPE, the damage is integrated into both the instantaneous and relaxed states of the DNLR model. Eq. 15 defines the damaged relaxed modulus ($E^{eff,r}$), which governs the material's stiffness at equilibrium. In contrast, Eq. 16 describes the kinetic evolution of the effective instantaneous modulus (E^{eff}) as a function of strain, capturing the physical degradation caused by cavitation.

$$E^{r,eff} = E^r \left[1 - \left[\alpha_1 \left(1 - e^{-\beta_1 \varepsilon_1} \right) - \frac{1}{5} \chi L^{-1} \left(\frac{\lambda_C}{n^{0.5}} \right) \varepsilon_1 \right] \right] \quad (15)$$

$$E^{eff} = E \left[1 - \left[\alpha_1 \left(1 - e^{-\beta_1 \varepsilon_1} \right) - \frac{1}{5} \chi L^{-1} \left(\frac{\lambda_C}{n^{0.5}} \right) \varepsilon_1 \right] \right] \quad (16)$$

The equilibrium stress was approached by the 8 chains model of Arruda and Boyce [17], which is based on the statistical theory related to molecular chains.

In the case of uniaxial solicitation (tensile compression), this stress evolution obeys to the following expression:

$$\sigma_1^{eq} = \frac{N_{rl} K_B T}{3\lambda_C} n^{0.5} \left(\lambda_1^2 - \lambda_2^2 \right) L^{-1} \left(\frac{\lambda_C}{n^{0.5}} \right) \quad (17)$$

$$\lambda_1 = \exp(\varepsilon_1) \quad (18)$$

$$\lambda_2 = \frac{1}{\sqrt{\lambda_1}} \quad (19)$$

$$\lambda_C = \sqrt{\lambda_1^2 + \frac{2}{3}\lambda_2^2} \quad (20)$$

The model flexibility of Arruda and Boyce [17] lies in the possibility of reproducing the hardening led by stretching and reorientation of chains, in large deformations, by bringing in

only two parameters n and NKT. These last ones allow finding the global shape of the hyperelastic hardening.

The corresponding good values (listed below in the table) of these parameters were obtained after several simulations.

Table 1: Value of material parameters for HDPE

E (MPa)	E^r(MPa)	n	NKT (MPa)	α_1	β_1
1180	450	90	0.53	0.899	1 8

The equilibrium stress is supposed to interpret a reversible thermodynamic state; the figures 1, 2 and 3 illustrate the evolution of equilibrium stress to axial strain, the number n of segments per chain and the rubbery modulus parameter (NKT).

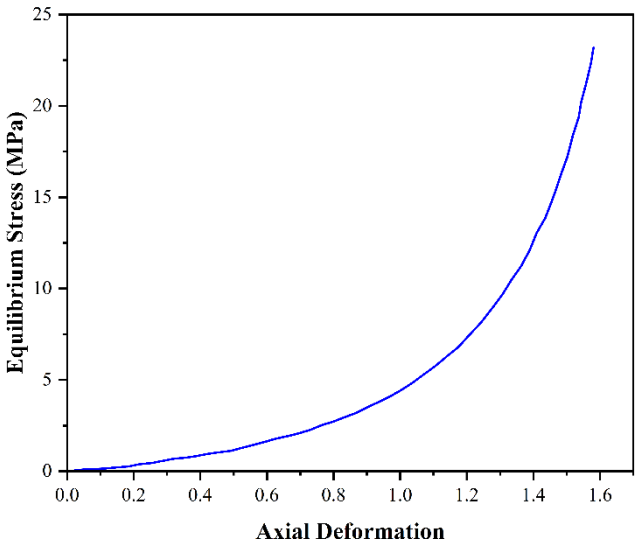

Figure 1: Evolution of equilibrium stress versus the axial strain

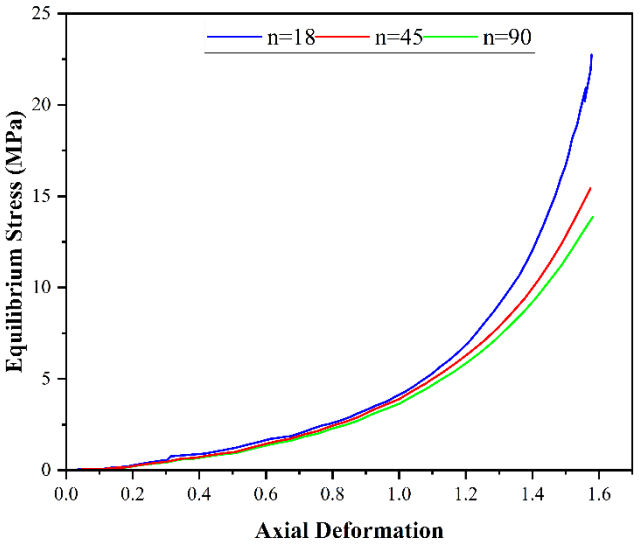

Figure 2: Influence of n on the equilibrium stress

Figure *2* illustrates the influence of the parameter (n) on the equilibrium stress response. According to the statistical theory of 8-chain networks, n dictates the limiting extensibility of the polymer chains. The results demonstrate that a lower value of (n) results in a higher stress response and a more rapid onset of hyperelastic hardening compared to higher values $(n = 90)$. This is physically consistent with the fact that shorter chains reach their maximum stretch more quickly, thereby increasing the material's resistance to further deformation.

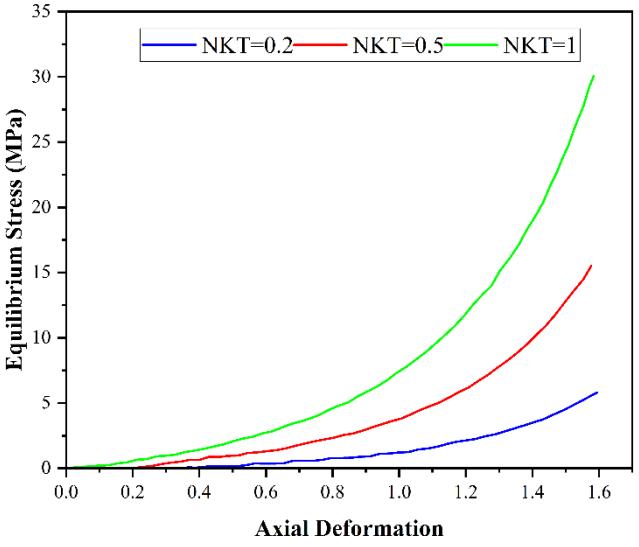

Figure 3: Influence of NKT on the equilibrium stress

The relaxation stress is defined by the equilibrium stress when $\tau^{eq} \ll \tau^{r}$. In figures 4, 5 and 6 demonstrate the influence of the relaxed Young's modulus (E_r), the chain segment parameter (n), and the damage magnitude coefficient (α_1) on the stress relaxation response, respectively.

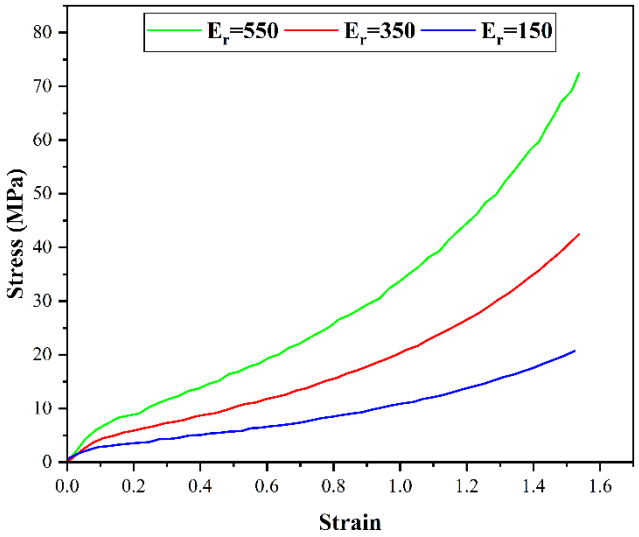

Figure 4: Influence of the relaxed modulus on the relaxed stress

Figure *4* depicts the sensitivity of the relaxation stress to variations in the initial relaxed Young's modulus (E_r). In the DNLR framework, E_r represents the stiffness of the material at the pseudo-equilibrium state once the viscous dissipative processes have subsided. The simulations show that a higher relaxed modulus ($E_r = 550\ MPa$) leads to a higher stress level across the strain range compared to a lower modulus ($E_r = 150\ MPa$). This alignment confirms that the stress response is directly proportional to the equilibrium stiffness of the macromolecular network, as required by thermodynamic consistency.

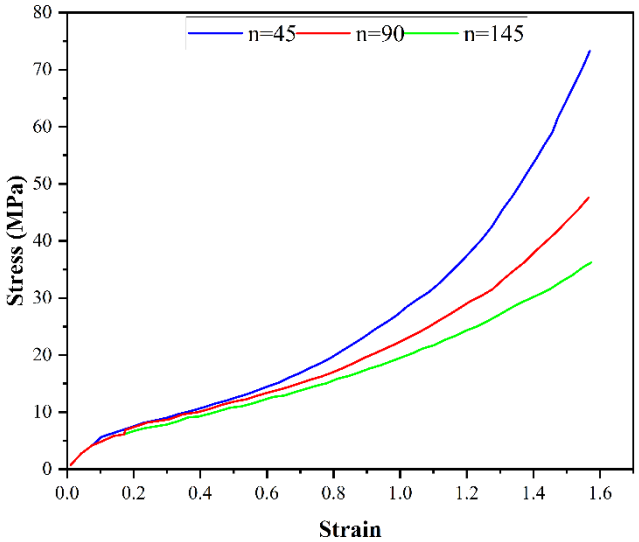

Figure 5: Effect of term n on the relaxed stress

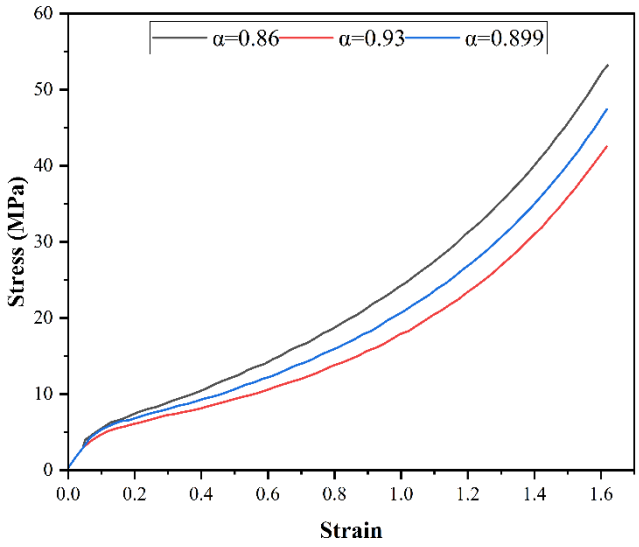

Figure 6: Effect of term α_1 on relaxed stress.

IV. Results and discussion

This study characterizes the evolution of damage in HDPE under uniaxial tension, with a specific focus on the variation of the Young's modulus as a function of strain [1], [8]. The results indicate that the apparent elastic modulus initially decreases sharply with axial deformation, reaching a limiting value of approximately 180 MPa due to the occurrence of cavitation damage [1]. Subsequently, the modulus enters a significant growth phase that progresses continuously until failure[6]. This recovery is attributed to the reorientation and stretching of macromolecular chains [2], [3], [6].

Despite the recovery of the Young's modulus at large strains, the value reached at failure does not return to its initial level. The numerical predictions of the proposed model demonstrate good agreement with the experimental results (see Fig. 7). A similar correlation is observed for the evolution of true stress as a function of strain (see Fig. 8).

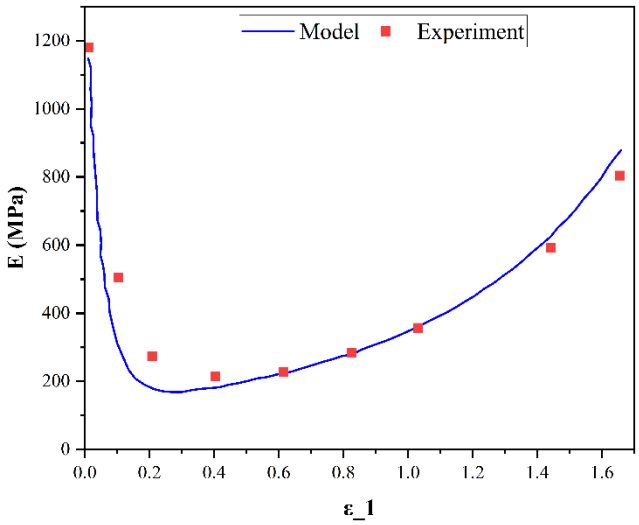

Figure 7: Evolution of the Young modulus, comparison of experimental and numerical results.

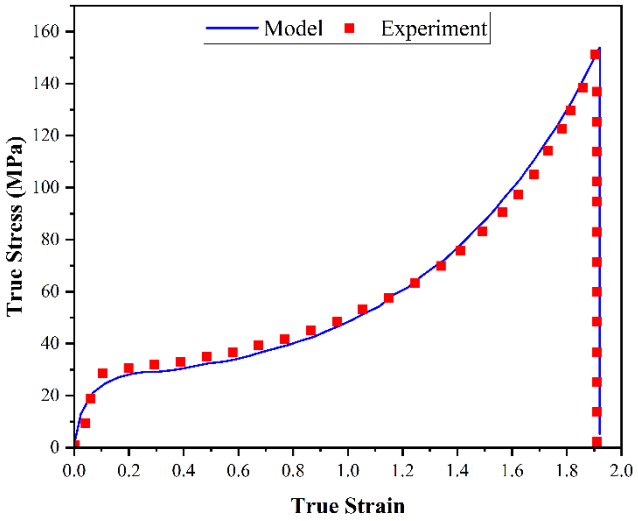

Figure 8: stress-strain curve, comparison of experimental and numerical results.

The quantitative assessment of the proposed DNLR formulation demonstrates a strong global reliability, as evidenced by a coefficient of determination (R^2) of 0.9486. This high value confirms that the model, when integrated with the Arruda and Boyce [17] eight-chain hyperelastic framework, successfully captures approximately 95% of the variance in the material's mechanical response during large plastic deformations [6]. The identification of precise material parameters was essential for this degree of correlation. The initial Young's modulus (E) of 1180 MPa represents the unrelaxed or instantaneous elastic response, while the relaxed Young's modulus (E^r) of 450 MPa reflects the pseudo-equilibrium stiffness of the material once viscous stresses have fully decayed [6].

Table 2: Statistical comparison numerical simulation vs experimental

$RMSE$	$RMSE(\%)$	MAE	MAE (%)	R^2
7.37	12.44%	3.108	5.24%	0.9486

The Mean Absolute Error (MAE) of 5.24% further validates the model's accuracy across the entire strain range, indicating that the numerical predictions closely follow the experimental stress-strain data [6], [8]. This average accuracy is largely supported by the model's ability to simulate hyperelastic hardening, which is governed by the number of rigid segments per chain ($n = 90$) and the rubbery modulus parameter ($NKT = 0.53MPa$) [6], [17]. The parameter is particularly critical as it dictates the limiting extensibility of the macromolecular network, marking the point where orientation-induced consolidation begins to significantly increase the material's resistance to further deformation [2], [3].

Localized deviations, reflected in the RMSE of 12.44%, likely occur during the complex transition between initial softening and final hardening, where localized phenomena like necking propagation are most intense [6], [16]. This phase is heavily influenced by the progression of internal damage, modeled here through the empirical parameters $\alpha_1 = 0.899$ and $\beta_1 = 18$. These coefficients effectively describe the total magnitude and the kinetics of cavitation and micro-void formation within the interlamellar amorphous domains [1], [6], [16]. Despite these minor fluctuations, the model accurately reproduces the characteristic "bathtub" evolution of the apparent elastic modulus, capturing the initial sharp stiffness loss down to a characteristic limiting value of approximately 180 MPa before the subsequent recovery induced by chain reorientation [1], [8].

V. Conclusions

In this work, we presented a constitutive framework based on the Distribution of Non-Linear Relaxation (DNLR) formalism, which establishes a rigorous link between the dissipative thermodynamics of relaxation in continuous media and the microscopic deformation physics of High-Density Polyethylene (HDPE). The primary contribution of this study is the development of a new model formulation that explicitly accounts for the evolution of internal damage and its coupling with the mechanical response.

The proposed model successfully reproduces the characteristic "bathtub" evolution of the apparent Young's modulus, capturing the sharp initial degradation down to a limiting value of approximately 180 MPa due to cavitation and micro-voiding, followed by a continuous recovery phase attributed to the reorientation and stretching of macromolecular chains. By integrating the statistical mechanics of the Arruda and Boyce (1993) eight-chain framework into the DNLR equilibrium state, the model provides a physically based description of hyperelastic hardening at large strains.

Numerical simulations demonstrated a high degree of quantitative accuracy, supported by a coefficient of determination (R^2) of 0.9486 and a Mean Absolute Error (MAE) of 5.24% when compared to experimental stress-strain data. The identified material parameters, including the initial modulus ($E = 1180MPa$), the number of segments per chain ($n = 90$), and the damage kinetics coefficients (α_1, β_1), are consistent with the known microstructural characteristics of semi-crystalline polymers. Ultimately, this unified thermodynamic approach proves to be a robust tool for predicting elastic-viscoplastic behavior and damage progression of thermoplastic polymers up to failure.

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***List of symbols**:*

$\boldsymbol{\sigma}$	*Observable mechanical stress.*
$\dot{\boldsymbol{\sigma}}$	*Stress rate.*
$\boldsymbol{\varepsilon}$	*True mechanical strain.*
$\dot{\boldsymbol{\varepsilon}}$	*strain rate.*
$\boldsymbol{A^j}$	*Thermodynamic affinity associated with the j^{th} relaxation mode.*
$\boldsymbol{z^j}$	*Internal variable representing the degree of advancement of the j^{th} dissipative process.*
$\boldsymbol{\psi_k}$	*Generalized thermodynamic potential (free energy).*
$\boldsymbol{T}$	*Absolute Temperature (°K).*
$\boldsymbol{s}$	*Specific entropy.*
$\boldsymbol{a^u}$	*Instantaneous stability tensor / the Tisza matrix.*
$\overline{\boldsymbol{b}}$	*Rectangular coupling tensor between reversible and irreversible state variables.*
$\boldsymbol{g}$	*Internal stability tensor / dissipation matrix.*
$\boldsymbol{\tau_j^r}$	*Reference relaxation time of mode in the relaxed state.*
$\boldsymbol{a(t)}$	*Time-dependent non-linearity factor / sliding factor.*
$\boldsymbol{P_0^j}$	*Statistical weight of the j^{th} relaxation mode.*
$\boldsymbol{E}$	*Initial Young's modulus.*
$\boldsymbol{E^r}$	*Initial Young's modulus at the relaxed state.*
$\boldsymbol{E^{eff}}$	*Effective (damaged) Young's modulus.*
$\boldsymbol{E^{eff,r}}$	*Effective Young's modulus at the relaxed state.*
$\boldsymbol{D}$	*Scalar damage variable.*
$\boldsymbol{\alpha_1}$	*Parameter characterizing the maximum damage level by cavitation.*
$\boldsymbol{\beta_1}$	*Parameter characterizing the damage saturation rate.*
$\boldsymbol{\chi}$	*Parameter related to macromolecular chain reorientation.*
$\boldsymbol{n}$	*Number of rigid segments per macromolecular chain.*
$\boldsymbol{NKT}$	*Rubbery modulus parameter related to chain density.*
$\boldsymbol{\lambda_c}$	*Internal stretch of a localized network chain.*
$\boldsymbol{\lambda_1, \lambda_2}$	*Principal stretch ratios.*
$\boldsymbol{L^{-1}}$	*Inverse Langevin function.*
$\boldsymbol{k_B}$	*Boltzmann constant.*

***References**:*

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