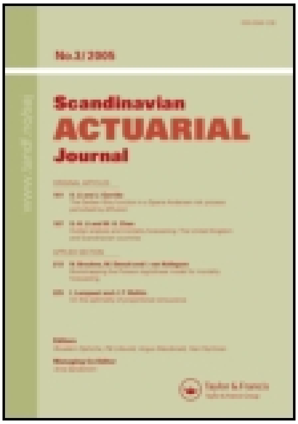

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Strong stability in a two-dimensional classical risk model with independent claims

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Original Article

Strong stability in a two-dimensional classical risk model with independent claims

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In this paper, we study the strong stability of ruin probabilities in risk models. The question of stability naturally arises in risk theory since the governing parameters in these models can only be estimated with uncertainty. Moreover, in most cases there are not explicit expressions known for the ruin probabilities. Our objective is to present the applicability of the strong stability method to the bivariate classical risk model with independent claims. After clarifying the conditions to approximate the two-dimensional risk model with disturbance parameters by the two-dimensional classical risk model, we obtain the stability inequalities with an exact computation of the constants.

Keywords: Risk models; Ruin probabilities; Markov chain; Strong stability; Reversed process; Continuity estimates

1. Introduction

In the actuarial literature, the evolution in time of the capital of insurance company is often modeled by the process of reserve resulting from the difference between the premium-income and the pay-out process (see Grandell (1991), Schmidli (1992)).

In the univariate classical risk model, one claim event yields one claim. Recently, multivariate risk models have also been introduced and studied in the literature (see Chan *et al.* (2003), and the references therein). The multivariate processes are introduced to describe insurance companies with more than one line of business.

The probability of ruin is one of the basic characteristics of risk models (see Asmussen (2000)). Since the first works of Lundberg (1926), Cramér (1930), Lundberg (1932) and Cramér (1955), various authors investigate the problem of its evaluation. We refer the reader for details to see Teugels (1982), Asmussen & Rolski (1991), Grandell (1991), Kalashnikov & Konstantinidis (1996), De Vylder *et al.* (1997) and Willmot & Lin (2001), but it cannot, however, be found in an explicit form for many risk models. Furthermore, parameters governing these models are often unknown and one can only give some bounds for their values. In such a situation the question of stability become crucial, see Beirlant & Rachev (1987) and Kalashnikov (1997).

In this work, we attempt to extend the investigation of the ruin probabilities realized by Kalashnikov (2000) in a univariate risk models to a bivariate risk process. So, we propose

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the application of the strong stability method to the two-dimensional classical risk model presented by Chan *et al.* (2003).

In the stability theory, we establish the domain within which a model may be used as a good approximation or idealization of the real system under consideration, see Zolotarev (1975), Borovkov (1984) and Rachev (1989).

In other words, we clarify here the conditions for which the proximity in one way or another of the parameters of the system involved the proximity of the studied characteristics. Such results give the possibility of approximating some systems complicated by other systems more exploitable or much simpler.

There exist numerous results on perturbation bounds of Markov chains. General results are summarized by Heidergott & Hordijk (2003). One group of results concerns the sensitivity of stationary distribution of a finite, homogeneous Markov chain (see Heidergott *et al.* (2007)), and the bounds are derived using methods of matrix analysis; see the review of Cho & Meyer (2001), recent papers of Kirkland (2002), and Neumann & Xu (2003). Another group includes perturbation bounds for finite-time and invariant distributions of Markov chains with general state space; see Aïssani & Kartashov (1983), Anisimov (1988), Rachev (1989), Kalashnikov & Konstantinidis (1996), Yu (2005), Bouallouche-Medjkoune & Aïssani (2006).

The strong stability method has been developed in the early 1980s by Aïssani and Karatshov (see Aïssani & Kartashov (1983), Kalashnikov & Konstantinidis (1996)). It allows both to make qualitative and quantitative analysis of some complex systems. This approach assumes that the perturbation of the transition kernel is small with respect to a certain norm (see Rabta & Aïssani (2005, 2008)). Such a strict condition allows us to obtain better estimations on the stationary characteristics of the perturbation chain. In addition, using this method, it is possible to obtain inequalities of stability with an exact computation of the constants.

After introducing the problems of stability in insurance mathematics by Beirlant & Rachev (1987), Kalashnikov (2000), using the strong stability method, investigated the estimation of ruin probabilities in the univariate risk models. Then, many authors; see Enikeeva *et al.* (2001), Rusaityte (2001a,b) extend the application of this approach for different type of risk process.

In a general form, this approach (Strong stability method) is based on the following three steps. The first step consists of identification of the ruin probability $\Psi_a(u)$ associate to risk model governing by a vector parameter a, with a stationary distribution for a specific random process which is called a reversed process. Such identification is well-known and it has been investigated in recent works by Asmussen & Kella (1996) and Asmussen & Sigman (1996).

Let us denote this reversed process by $V(t,a)$. The identification mentioned above means that:

$$\Psi_a(u) = \lim_{t \to \infty} \mathbb{P}(V(t,a) > u)$$

where u is the initial reserve.

The second step consists of the embedding $V(t,a)$ into a Markov process by equipping it with supplementary coordinates. So, we consider an enlarged process $W(t,a) = (V(t,a), Y(t,a))$ which is Markov.
In this terms,

$$\Psi_a(u) = \pi_a(\Gamma_u)$$

where π_a is stationary distribution of the Markov process $W(t,a)$ and $\Gamma_u = \{W = (V,Y), V > u\}$.

The third step consists of the application of the quantitative aspect of this method, giving the possible deviations of stationary distributions of the two Markov process under comparison governed by parameters a and a', respectively.

The rest of this paper is organized as follows. In Section 2, we introduce the proposed risk model and some basic assumptions. Section 3 concern the study of the strong stability of one type of ruin probabilities defined in Section 2. In Section 4, we clarify the strong stability conditions to obtain quantitative estimates which serve for delimiting domain where the two-dimensional classical risk model can be a good approximation of another disturbance two-dimensional risk model and estimate the error of approximation with respect to the norm $\|\cdot\|_{v}$. Finally, in Section 5, we present our conclusion.

2. The two-dimensional classical risk model

Consider an insurance company with two line of business. The evolution in time of the capital of this company is often modeled by the process of reserve $\{X(t), t \geq 0\}$ described by:

$$X(t) = \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} t - \sum_{j=1}^{N(t)} \begin{pmatrix} Z_j^1 \\ Z_j^2 \end{pmatrix} \qquad t \geq 0 \tag{1}$$

Where the initial reserve u_i and the gross premium rate c_i, $i = 1,2$ are real positive constants and $N(t)$ be the number of claims between time 0 and t, which follows a Poisson process with parameter λ.

For fixed $i = 1$ or 2, $\{Z_j^i, j = 1,2,\ldots\}$ are independent and identically distributed claim size random variables with common distribution F_i. For simplicity, we assume that $\{Z_j^1, j = 1,2,\ldots\}$ and $\{Z_j^2, j = 1,2,\ldots\}$ are independent and furthermore, both of them are also independent of $\{N(t), t \geq 0\}$.

In addition, the corresponding relative security loading vector is denoted by $\eta = (\eta_1, \eta_2)$ with

$$\eta_i = \frac{c_i}{\lambda \mu_i} - 1 > 0$$

and $\mu_i = E[Z_j^i]$, $i = 1$ or 2.

The concept of 'ruin' in multi-dimensional cases could have different meanings and interpretations when compared to the standard univariate risk process. In this paper, we consider the following three types of time of ruin:

$$T_{\min} = \inf\{t/\min(X_1(t), X_2(t)) < 0\} \tag{2}$$

$$T_{\max} = \inf\{t/\max(X_1(t), X_2(t)) < 0\} \tag{3}$$

and

$$T_{sum} = \inf\{t/X_1(t) + X_2(t) < 0\} \tag{4}$$

Various ruin probabilities in multivariate risk models are often of fundamental interest in risk management.

With the time of ruin defined, the corresponding probability of ruin is denoted by

$$\Psi_{\min}(u_1, u_2) = \mathbb{P}(T_{\min} < \infty/(X_1(0), (X_2(0)) = (u_1, u_2)) \tag{5}$$

$$\Psi_{\max}(u_1, u_2) = \mathbb{P}(T_{\max} < \infty/(X_1(0), (X_2(0)) = (u_1, u_2)) \tag{6}$$

and

$$\Psi_{sum}(u_1, u_2) = \mathbb{P}(T_{sum} < \infty/(X_1(0), (X_2(0)) = (u_1, u_2)) \tag{7}$$

Notice that $T_{\min}$, $T_{\max}$ and T_{sum} have natural interpretations, for instance, $\{T_{\min} < \infty\}$ means that at least one of $\{X_i(t),\ i = 1,2\}$ is below zero in the future, $\{T_{\max} < \infty\}$ means that both $X_1(t)$ and $X_2(t)$ are below zero at the same time t in the future, and $\{T_{sum} < \infty\}$ implies that the total of $X_1(t)$ and $X_2(t)$ is negative for one or more times in the future.

A two-dimensional case of the multivariate compound Poisson risk model has been discussed by Chan *et al.* (2003), in which, the claim sizes Z_j^1 and Z_j^2 are assumed to be independent. They obtained an explicit formula for Ψ_{sum} for the risk models with the claims of phase type.

The three types of ruin probabilities Ψ_{min}, Ψ_{max} and Ψ_{sum}, related to the bivariate risk model are the usual studied in the literature. However, Ψ_{sum} is more significant because it provides useful information to the management of the insurance company. So, in this paper, we are interested in the ruin probability Ψ_{sum}.

3. Strong stability in two-dimensional classical risk model

3.1. Preliminaries and notations

Let $m\mathcal{E}$ be the space of finite measures on the probabilizable space $(E, \mathcal{E})$, and $f\mathcal{E}$ the space of bounded measurable function on E. We associate with each transition kernel P the linear mapping:

$$\mu P(A) = \int_E \mu(dx) P(x, A),\ \forall\ \ A \in \mathcal{E} \tag{8}$$

$$Pf(x) = \int_E P(x, dy) f(y),\ \forall\ \ x \in E \tag{9}$$

Introduce on $m\varepsilon$ the class of norms of the form

$$\|\mu\|_v = \int_E v(x)|\mu|(dx) \tag{10}$$

Where v is an arbitrary measurable function (not necessarily finite) bounded below away from a positive constant, and $|\mu|$ is the variation of the measure μ.

This norm induces in the space $f\varepsilon$ the norm:

$$\|f\|_{\upsilon} = \sup\{|\mu f|, \ \|\mu \mathrm{f}\|_{\upsilon} \leq 1\} = \sup_{x\in E}[\upsilon(x)]^{-1}|\mathrm{f}(x)|, \forall f \in f\mathcal{E} \tag{11}$$

Let us consider $\mathcal{B}$, the space of linear operators, with the norm:

$$\|P\|_{\upsilon} = \sup_{x\in E}\left([\upsilon(x)]^{-1}\int_{E}\upsilon(y)|P(x,dy)|\right) \tag{12}$$

Definition 3.1 A Markov chain X with a transition kernel P and invariant measure π is said to be strongly υ– stable with respect to the norm $\|\cdot\|_{v}$, if $\|P\|_{v} < \infty$, and each stochastic kernel Q on the space in some neighborhood $\{Q{:}\|Q-P\| \ < \varepsilon\}$ has a unique invariant measure $v = v(P)$ and $\|\upsilon{-}\pi\|_{\upsilon} \to 0$ as $\|Q-P\|_{\upsilon} \to 0$.

In the sequel, we use the following results:

Theorem 3.1 *The Markov chain X with the transition kernel P and invariant measure* π *is strongly* υ*-stable with respect to the norm* $\|\cdot\|_{\upsilon}$*, if and only if there exist a measure* α *and a non-negative measurable function h on E such that* $\pi h > 0$, $\alpha\mathbf{1} = 1$, $\alpha h > 0$ (see Aïssani & Kartashov (1983)), and

- The operator $T = P{-}\ h\circ\alpha$ is non-negative.
- There exist $\rho < 1$ such that $T\upsilon\ (x) \leq \rho\upsilon\ (x)$ for $x \in E$.
- $\|P\|_{\upsilon} < \infty$.

Here **1** is the function identically equal to 1 and $\circ$ denotes the convolution between a measure and a function.

The following result was proved by Kartashov (1996).

Theorem 3.2 *Let v be the fixed weight function and assume that a Markov chain with the transition probability P, satisfying* $\|P\|_{\upsilon} < \infty$*, possess a unique stationary distribution* π*. Assume also that there exist a non-negative function h and a probability measure* α *such that P can be splitted as follows* (*see Kalashnikov (2000)*):

$$P(x,.) = T(x,.) + h(x).\alpha(.) \tag{13}$$

Where

$$\|\pi\|_{h} > 0, \ \|\alpha\|_{h} > 0 \tag{14}$$

and

$$\|T\|_{\upsilon} \leq \rho < 1 \tag{15}$$

Then each Markov chain with the transition probability P' *satisfying the inequality*

$$\Delta = \|P - P'\|_{v} < \Delta_0 \equiv \frac{(1-\rho)^2}{1-\rho+\rho\|\alpha\|_{v}} \tag{16}$$

has a unique stationary π' *and, furthermore*

$$\|\pi - \pi'\|_v \leq \frac{\Delta\|\alpha\|_v}{(1-\rho)(\Delta_0 - \Delta)} \tag{17}$$

3.2. Strong stability

This part concerns the study of strong stability of the reversed process associate to our model.

Since the ruin can only happen at claim occurrence times $\{T_n\}$, the probability of ruin $\Psi_{sum}(u_1,u_2)$, defined by the relation (7), can be expressed in terms of the process $\{X^1_{T_n}\}$ and $\{X^2_{T_n}\}$ as:

$$\Psi_{sum}(u) = \mathbb{P}\left(\inf_{n\geq 1}(X^1_{T_n} + X^2_{T_n}) < 0/X^1_0 + X^2_0 = u\right) \tag{18}$$

where $u = u_1 + u_1$ and $\{T_n, n \geq 1\}$ be a successive i.i.d. occurrence times. The reversed process $\{V_n\}_n$ associate to our risk model can be defined by the equation:

$$\forall n \geq 0,\ V_{n+1} = (V_n - (c_1 + c_2)\theta_{n+1} + Z^1_{n+1} + Z^2_{n+1})_+,\ V_0 = 0 \tag{19}$$

With $T_n = \theta_1 + \theta_2 + \cdots + \theta_n$ and θ_n be successive i.i.d inter-occurrence times.

According to the recursive form of the reversed process $\{V_n\}_{n\geq 0}$, we have that V_{n+1} depend only on V_n, θ_{n+1}, Z^1_{n+1} and Z^2_{n+1} where the random variables θ_{n+1}, Z^1_{n+1} and Z^2_{n+1} are independent on n and on the state of the system before n.

Then $\{V_n\}_{n\geq 0}$ is a homogenous Markov chain and its state space is $E = \mathbb{R}^+$. Denote by

$$P(x, A) = \mathrm{P}(V_{n+1} \in A/V_n = x) \tag{20}$$

its transition probability.

Using this Markov chain $\{V_n\}_{n\geq 0}$, it is well-known that

$$\Psi_{sum}(u) = \lim_{n\to\infty} \mathbb{P}(V_n > u) \tag{21}$$

Remark 3.1 *The aim of this paper is to obtain continuity estimates for the ruin probability Ψ_{sum} by using the strong stability approach, which consists in some steps. The first one concerns the identification of the ruin probability with a stationary probability of a specific random process which is a reversed process (homogeneous Markov chain). General constructions of reversed processes can be found in Asmussen & Kella (1996), Asmussen & Sigman (1996) and Enikeeva et al. (2001). These constructions are purely algebraic and do not use probabilistic structure. However, the processes $\{\overline{X}(t),\ t\geq 0\}$ and $\{\overline{\overline{X}}(t), t\geq 0\}$ defined by $\overline{X}(t) = min(X_1(t), X_2(t))$ and $\overline{\overline{X}}(t) = max(X_1(t), X_2(t))$ are not markovian and the construction of the corresponding markovian processes are not easy to do. Note that there are no explicit expressions for Ψ_{min} and Ψ_{max} even in case of exponential distribution claims size. Nevertheless, we think that using the theory of regenerative processes, we could obtain some continuity estimates for this two ruin probabilities.*

3.3. Transition kernel

The transition kernel associate to the chain $\{V_n\}_{n\geq 0}$ defined on the probabilisable space $(E, \mathcal{E})$ can be split as follows:
$\forall\, x \in \mathbb{R}^+$ and $\forall\, A \in \mathcal{E}$, we have:

$$\begin{aligned} P(x, A) &= \mathbb{P}(V_1 \in A / V_0 = x) = \mathbb{P}((V_0 - (c_1 + c_2)\theta_1 + Z_1^1 + Z_1^2)_+ \in A / V_0 = x) \\ &= \mathbb{P}(0 < (x - (c_1 + c_2)\theta_1 + Z_1^1 + Z_1^2) \in A) \\ &\quad + \mathbb{P}(0 \in A)\mathbb{P}(x - (c_1 + c_2)\theta_1 + Z_1^1 + Z_1^2) \leq 0) \\ &= T(x, A) + \alpha(A).h(x) \end{aligned} \tag{22}$$

With

$$T(x, A) = \mathbb{P}(0 < (x - (c_1 + c_2)\theta_1 + Z_1^1 + Z_1^2) \in A), \;\; \alpha(A) = \delta_0(A)$$

where δ_0 is a probability measure concentrated at 0 (Dirac measure), and

$$h(x) = \mathbb{P}((c_1 + c_2)\theta_1 - (Z_1^1 + Z_1^2) \leq x), \;\; x \in \mathbb{R}^+.$$

To apply the Theorem 3.1 to the Markov chain $\{V_n\}_{n\geq 0}$, we choose the function $v(x) = e^{\varepsilon x}$, $x \in \mathbb{R}^+$. All conditions of this theorem are satisfied for:

- $T(x, A) = \mathbb{P}(0 < (c_1 + c_2)\theta_1 + Z_1^1 + Z_1^2) \in A)$.
- $\alpha(A) = \delta_0(A)$ (Dirac measure).
- $h(x) = \mathbb{P}((c_1 + c_2)\theta_1 - (Z_1^1 + Z_1^2) \leq x), \;\; x \in \mathbb{R}^+$.

obtained by the precedent decomposition of the transition kernel P.
Where

$$\rho = \mathbb{E}(\exp\{\epsilon(Z_1^1 + Z_1^2 - (c_1 + c_2)\theta_1)\}) \tag{23}$$

Finally, the Markov chain $\{V_n\}_{n\geq 0}$ is strongly stable for the weight function $\upsilon(x) = e^{\epsilon x}$, $x \in \mathbb{R}^+$.

Then we have the following result.

Theorem 3.3 *Consider the two-dimensional classical risk model with independent claims. Then, there exist $\varepsilon > 0$ such that the reversed process $\{V_n\}_{n\geq 0}$ (Markov chain) associate to this model is strongly stable with respect to the weight function $\upsilon(x) = e^{\epsilon x}$, $x \in \mathbb{R}^+$.*

Let us now illustrate how Theorem 3.2 can be applied to obtain stability bounds.

4. Stability inequalities

In this section, we are interested to obtain quantitative estimates which serve for delimiting domain where the two-dimensional classical risk model can be a good approximation of another disturbance two-dimensional risk model and to estimate the error of approximation.

The two-dimensional classical risk model is completely determined by the vector of parameters $a = (c,\lambda,F_1,F_2)$. As we have seen, the probability of ruin $\Psi_{sum}(u)$ coincides with the stationary distribution of the reversed process $\{V_n\}_{n\geq 0}$ (see Kartashov (1996)) to exceed the level u.

Let $a' = (c',\lambda',F'_1,F'_2)$ be the vector parameter governing another bivariate risk model, its ruin probability being $\Psi'_{sum}(u)$ and $\{V'_n\}_{n\geq 0}$ its reversed process associate.

4.1. Estimation of the transition kernel deviation

To be able to estimate numerically the margin between the stationary distributions of the Markov chains $\{V_n\}_{n\geq 0}$ and $\{V'_n\}_{n\geq 0}$, we estimate the norm of the deviation of transition kernel.

According to (Kalashnikov (2000)), the deviation $\|P - P'\|_v$ can be estimated as follows:

$$\|P - P'\|_v \leq 2\mathbb{E}\ e^{\epsilon}(Z_1 + Z_2)\ \ln\left|\frac{\lambda(c'_1 + c'_2)}{\lambda'(c_1 + c_2)}\right| + \|F_1 * F_2 - F'_1 * F'_2\|_v \tag{24}$$

Where $F_1 * F_2$ is the distribution function of the random variable $(Z_1 + Z_2)$ which is sum of independent random variables. Also for $F'_1 * F'_2$

Another estimation of the transition kernel deviation can be deducted from the paper of (Enikeeva *et al.* (2001)).

4.2. Stability inequalities

This subsection consists to determinate the ruin probabilities deviation with respect to the norm $\|.\|_v$.
Denote

$$\mu(a,a') = 2\mathbb{E}\ e^{\epsilon}(Z_1 + Z_2)\ \ln\left|\frac{\lambda(c'_1 + c'_2)}{\lambda'(c_1 + c_2)}\right| + \|F_1 * F_2 - F'_1 * F'_2\|_v$$

Under assumption $\mu(a, a') < (1-\rho)^2$ and from inequality (17) of Theorem 3.2, the distance between ruin probabilities is expressed as follows:

$$\|\psi_{sum}(u) - \psi'_{sum}(u)\|_v \leq \frac{\mu(a,a')}{(1-\rho)((1-\rho)^2 - \mu(a,a'))} \tag{25}$$

where ρ is given by relation (23).

Then, we have the following result.

Theorem 4.1 *Let $\Psi_{sum}(u)$ and $\Psi'_{sum}(u)$ the ruin probabilities associates to the reversed processes $\{V_n\}_{n\geq 0}$ and $\{V'_n\}_{n\geq 0}$ respectively. Then, under assumption*:

$$\mu(a,a') < (1-\rho)^2 \tag{26}$$

we have

$$\|\psi_{sum}(u) - \psi'_{sum}(u)\|_{\upsilon} \leq \frac{\mu(a,a')}{(1-\rho)((1-\rho)^2 - \mu(a,a'))} \tag{27}$$

and

$$\rho = \mathbb{E}\left(exp\{\epsilon(z_1^1 + z_1^2 - (c_1+c_2)\theta_1)\}\right).$$

Remark 4.2 *In the case of a multidimensional classical risk model with k line of business (k >2), the corresponding process of reserve is defined by the following relation*:

Let us preserve the notations in section 2.

$$X(t) = \begin{pmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ Xt(t) \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_k \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{pmatrix} t - \sum_{j=1}^{N(t)} \begin{pmatrix} Z_j^1 \\ Z_j^2 \\ \vdots \\ Z_j^k \end{pmatrix} \qquad t \geq 0 \tag{28}$$

Under assumption of independent claims, the result obtained in this paper can be generalized to the case of multidimensional classical risk process with the same type of ruin probability Ψ_{sum}. Then continuity estimates can be obtained by considering the reversed process $\{V_n\}_{n\geq 0}$ associate to this ruin probability, defined by the equation

$$V_{n+1} = \left(V_n - (c_1 + c_2 + \ldots + c_k)\theta_{n+1} + \sum_{i=1}^{k} Z_{n+1}^i\right)_+, \qquad V_0 = 0 \tag{29}$$

Using this Markov chain $\{V_n\}_{n\geq 0}$, it is well-known that

$$\psi_{sum}(u) = \lim_{n\to\infty} \mathbb{P}(V_n > u) \tag{30}$$

Where $u = u_1 + u_2 + \ldots + u_k$.

5. Conclusion

In this work, we proved the applicability of the strong stability method to approximate one type of ruin probability in the two-dimensional classical risk model with independent claims.

The stability bounds of ruin probabilities derived above contain only explicitly written parameters. The precision obtained allows us to confirm the efficiency of this method and its importance for practical problems.

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